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TANK FIRE CONTROL SYSTEMS STUDY

EVALUATION OF SOME ALTERNATIVE SYSTEMS OF TANK STABILIZATION

BY
PHILIP I. BROWN

Ordnance Project TT2-693 DA Project 548-19-005

Fire Control Instrument Group
FRANKFORD ARSENAL
PHILADELPHIA, PA.

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EVALUATION OF SOME ALTERNATIVE SYSTEMS OF TANK STABILIZATION

ORDNANCE PROJECT TT2-693 FIRE CONTROL PROJECT 429

PREPARED BY:
PHILIP I. BROWN

APPROVED BY:
HARRY M. MURRAY
Colonel, Ord. Corps

FOR:

JOSEPH M. COLBY
Brigadier General, U.S.A.
Commanding

Fire Control Instrument Group, Frankford Arsenal, Philadelphia 37, Pa.
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ORDNANCE PROJECT TT2-693 FIRE CONTROL PROJECT 429

OBJECT

An investigation of various aspects of the tank stabilization problem was conducted to provide recommendations to higher echelons of the Ordnance Corps for specific adoption of an optimum stabilization system. This study evaluated the relative effects of separate stabilization of gun and sight versus combined stabilization, as well as the effects of varying degrees of stabilization. A scheme known as the three-switch proposal, which arose during the study, was also investigated. This plan is based upon the use of three switches in series, which operate automatically to prevent the gun from firing until there is sufficient probability of a hit.

SUMMARY

This report evaluates the relative merits, as they affect the time to fire the first shot and the single-shot hit probability, of the following systems of tank stabilization: (1) the gun and sight stabilized as a unit; (2) the gun and sight stabilized separately, with various degrees of stabilization; (3) each of these systems in conjunction with the three-switch proposal. In addition, this report proposes to utilize the mathematical model of the tank duel as a device for relating the time to fire and the accuracy of fire to the measure of effectiveness, i.e., probability of surviving a battle. Examples of some simplified duels and their uses are given.

In order to arrive at the single-shot hit probability, an error analysis of each system is made in such a manner as to enable the use of available data. For this purpose, the total error is considered to be composed of four components as follows: (1) the inherent accuracy of the system, (2) the ranging error, (3) the error due to the moving sight, and (4) the error due to the moving gun. It is shown that the total error is normally distributed with a mean of zero and a variance equal to the sum of variances of the component errors.

Data is presented for the variance of the component errors, which is based on experimental data with the exception of that due to the movement of the sight. Single-shot hit probability curves are then presented for the various systems under consideration and for various combinations of magnitudes of the component errors.

As a by-product of the above error analysis, it is shown that neither the burst-on-target method nor the use of a range finder is feasible in moving fire.

SUMMARY

An examination of the probability of hit data shows that worthwhile increases in the probability of hit may be obtained by a separate and more tightly stabilized sight, provided that the standard deviation of the error due to gun movements can be confined to values of approximately 1 to 1½ mils. While it is not considered feasible to accomplish such small errors due to gun movement by stabilization alone, it is shown that the three-switch proposal can accomplish this aim in a very economical fashion. It is further shown that the increase in time to fire the first shot, which is due to the adoption of a three-switch mechanism, is well within acceptable bounds.

There is some evidence that the above conclusions may be carried even further so that a final system would consist of a separately stabilized sight, very limited gun stabilization, and a three-switch mechanism.

Because of the preliminary nature of this report and the several items requiring further investigation, the above conclusions can be accepted only tentatively. These are, however, of sufficient promise to warrant further study.

AUTHORIZATION

ORDTT: OCM Items 31340, 31415, 31059, 32855; RAD Orders 1-12461, 3-3788; 00 121.2/4473.

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EVALUATION OF TANK STABILIZATION SYSTEMS

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EVALUATION OF SOME ALTERNATIVE SYSTEMS OF TANK STABILIZATION

I. INTRODUCTION

The Problem

During 1951 and 1952 the Fire Control Instrument Group, Frankford Arsenal, investigated various aspects of the tank stabilization problem. Specific evaluations were made of the system in which the gun and sight are independently stabilized and of the system in which only the sight is stabilized. In conjunction with each of these systems, the so-called three-switch proposal was evaluated.

The three-switch proposal centers about the operation of three switches: one is thrown by the gunner when he wishes to fire; the other two switches, one for azimuth and one for elevation, are thrown automatically when the gun comes sufficiently close to "on target" position. When all three switches are "on" at the same time, the gun will fire. The effect of the system is to keep the gun from firing when it is too likely to miss. The interrelated properties of time delays in firing and accuracy under this proposal are studied in this report.

Limitations of the Problem

The approach to this analysis has been to consider only the merits of such systems, the prior assumption being made that they could be built. No attempt has been made to consider the detailed engineering of the equipment involved,[1] and design has been discussed only: (1) when necessary from the viewpoint of the analysis, or (2) when some criteria for optimum design would result as a by-product of the systems evaluation.

This is a preliminary report in which very refined or elaborate evaluation is not intended. The various shortcomings of the analysis are discussed in the body of the text.

It is expected that further study will be of great value. However, since the cost of such study is high, it was felt that preliminary results should be reported here so that the project may be re-evaluated in the light of these findings. In general, this report is not intended to evaluate the usefulness of stabilization per se; it is rather intended to evaluate some alternative systems of stabilization on the assumption that a system exists which is worthwhile.[2]

[1] Subsequent to the completion of this report, the author became aware of another report of interest to this problem: "Director Type Tank Stabilization," Ordnance Division, Minneapolis-Honeywell Regulator Co., 16 June 1953, CONFIDENTIAL. (Ref. 1). This report discusses the engineering aspects of the problem.

[2] In the course of this investigation and the examination of other reports, some doubt arises that the more basic question of the advisability of any kind of stabilization has been sufficiently investigated.

A Note on Accuracy of Results and Sources of Data

Because of the preliminary nature of this report and the laboriousness of much of the calculation, the time consumed in checking the accuracy of calculations had to be held to a minimum. Due caution, of course, was exercised, and all calculations were examined for reasonableness of results. In addition, gross checks and cross checks were used.

The reader's attention is particularly directed to the sampling errors inherent in the "monte carlo" solution. The magnitude of these errors is dependent on the number of random selections used in the calculations (which are given in appropriate footnotes).

Data from various other reports which are quoted and used in this report are subject to the following limitations: (1) as in all such studies, the data selected for presentation represent the author's views as to the validity and appropriateness of the various available data; (2) not all data on test results is readily available, so that some better data may exist of which the author is unaware; and (3) since continual development and testing is going on in this field, additional (and perhaps better) information may be available after this report has been completed.

The Moving Fire Problem—Time vs Accuracy

The obvious purpose of stabilization of gun and/or fire control on a tank is to permit the tank to shoot with reasonable accuracy while it is on the move. There are at least two situations in which such a facility is considered desirable: (1) in an engagement with an anti-tank weapon, and (2) in a coordinated tank-infantry assault upon an enemy. In each of these roles, the advantage of time is gained by stabilization. Of course, the non-stabilized tank can perform the same functions of defending itself and aiding in an infantry assault merely by stopping to shoot. This, however, is considered a disadvantage in each of the above roles since: (1) in a tank to anti-tank battle, the time required for the tank to stop (while attempting to defend itself) *may* enable the enemy to get the first shot and thus materially lessen survival chances, and (2) the action of periodically stopping to fire during a tank-infantry assault may detrimentally affect the chances of success of the assault by slowing it down and reducing the shock effect of the tanks on enemy troops. Clearly, then, the advantage of time, with the correlative ability to fire more quickly, is what one aims to achieve by tank stabilization.[3]

Unfortunately, this advantage is not gained without the sacrifice of another important factor—accuracy of fire. The movement of the tank will cause perturbations of the gun and the fire control. These perturbations will, in turn, result in greater shot dispersion and reduction in probability of a target hit.

Therefore, a compromise is involved. One desires stabilization in order to gain the advantage of a *first shot*, but in order to gain this advantage it is necessary to suffer a disadvantage of a less accurate first shot. In considering a tank-to-tank battle, time to fire the first shot—along with accuracy—has influence on the probability of killing the enemy tank (or surviving the battle).

A central point of the tank stabilization problem, then, is the amount of accuracy one can afford to sacrifice in order to gain the advantage of firing the first shot.[4]

[3] Decreased vulnerability owing to ability to maneuver is, of course, an obvious additional advantage.
[4] See Section VIII of this report.

Some Published Data on Time and Accuracy in Moving Fire

The effects of moving fire on accuracy were brought out by the Armored Medical Research Laboratory, Fort Knox, Kentucky, in 1944.[5] A summary of these results is given in Table 1.

TABLE 1.

Standard Deviation (in mils) of Shots for Stationary Fire, Moving Fire with Gyrostabilizer, and Moving Fire without Gyrostabilizer—at 500 yd. Range. (Data from Ref. 2.)

	Standard Deviations	
	Azimuth: σ_x	Elevation: σ_y
Stationary Fire	0.21	0.17
Firing from a halt	0.75	0.90
Moving fire with gyrostabilizer	2.86	2.20
Moving fire without gyrostabilizer	2.80	6.00

The peculiarities of the test from which this data is taken have great influence on the actual values, but the figures in Table 1 are comparable among themselves and well illustrate the order of magnitude of accuracy lost with moving fire over firing from a halt, and the gains of stabilized moving fire over non-stabilized moving fire.

On the question of time gained, only slight gains in "rounds per minute" are shown in the Fort Knox report, Ref. 2. However, the experiment is somewhat biased toward this conclusion. More reliable figures on time are given on page 57 of this report.

The data given in the referenced report applies to firing trials on the M4A2 tank, which had stabilization in elevation only. Since the date of this report various other firing trials have been conducted with more recent equipment. A summary of the results of these trials is reproduced in Table 2. It is felt, however, that the ways in which the tests were conducted are sufficiently different that comparison of the results of one trial with those of another is not warranted. It is the opinion of the writer, based on an examination of these reports, that about the most that can be said for this data is that: (1) stabilization in azimuth is worthwhile if stabilization is at all worthwhile, and (2) some improvement of unknown magnitude in accuracy for elevation has resulted with the newer stabilization equipment.

Stabilization in Moving Fire

The function of stabilization is to keep the gun and the fire control in a relatively stable position in space, even though the rest of the tank is affected by perturbations induced by its motion. Just how stable it is possible to keep the gun and fire control depends, to a large extent, on how much power is available to perform the stabilization function, and the frequency and amplitude of induced perturbations. The perturbations, in turn, will depend upon the speed of the tank, the character of the terrain, and some of the characteristics of the tank, such as the weight and the suspension system. In the current type of arrangement one may hope to improve the accuracy of fire by attempting

[5] "Capacities and Limitations of Moving Fire with Gyrostabilizer," Armored Medical Research Laboratory, Fort Knox, Kentucky, 24 May 1944, RESTRICTED, (Ref. 2).

TABLE 2.
Standard Deviation (in Mils) of Rounds from Mean Center of Impact—Mean Range 1000-1200 yard.

Tank	Stabilizer	Course	Speed	σ_x Azimuth	σ_y Elevation
M4A3	"Standard"	Straight	10	2.45	1.64
M4A3	IBM	Straight	6	.90	1.13
		Zigzag	6	1.28	1.39
		Circular	6	1.72	1.58
M24	IBM	Straight	10	.86	.55
Centurion II	Metro-Vickers	Straight	6	.70	.62
		Straight	15	.59	1.23
		Zigzag	6	1.03	.78
Mk III		Circular	6	2.37	1.23
Centurion III	Metro-Vickers	Straight	10	1.47	1.47
		On Road	16	1.16	1.16
Mk IV		Zigzag	12	1.84	1.84
		Diagonal	12	2.09	2.09
Centurion III	Metro-Vickers	Straight (smooth)	6	.70	.64
		Straight (smooth)	15	1.25	1.44
Mk IV		Straight (rough)	6	.73	.69
		Straight (rough)	15	1.04	1.73
		Zigzag	6	1.24	.80
		Zigzag	15	2.22	1.30
		Circular	6	.70	.72
T41	Vickers	Straight (smooth)	6	.86	1.46
		Straight (smooth)	15	1.15	1.13
		Straight (rough)	6	1.63	2.14
		Straight (rough)	15	2.86	3.17
		Zigzag	6	1.93	2.37
		Circular	6	2.04	1.64

51. Data provided by Aberdeen Proving Ground based mainly on various unpublished test results.

to reduce the perturbations of gun and fire control (which are both stabilized together, and hence to the same degree of tightness). To date such attempts have not been as satisfactory as desired, partly because of the limited amount of power available for stabilization. Additional power cannot be made available without drastically increasing the size, cost, and complexity of the equipment.

As a means of reducing the total firing error of stabilized fire while not substantially increasing the power requirements, two proposals have been made:

(1) To stabilize the fire control independently from the gun. The basis of this proposal is that, because of the much smaller mass of the fire control, it can be tightly stabilized if done so separately from the gun. Since the total error in firing is thought to be a function of errors due to perturbations of fire control,[a] it is expected that the

[a] As well as errors due to gun perturbations.

reduction in fire control errors, resulting from tighter stabilization of fire control, will reduce the total firing error.

(2) To restrict the ability of the gun to fire so that it can fire only when it is sufficiently close to "on target" position.[7] When this type of firing system (called the three-switch proposal) is used, one gives up some of the time advantage gained by stabilization in order to reduce some of the inaccuracy disadvantage which results from firing on the move, but retains the advantage of reduced vulnerability due to movement.

It is the purpose of this report to evaluate the merits of these two proposals as compared with the current stabilization system.

As will be seen, the combination of more closely and independently stabilizing the sight and the three-switch firing system brings about an increase in accuracy without too much of a sacrifice in time to fire. In fact, it will be shown that a closely and independently stabilized sight and a three-switch firing arrangement without any gun stabilization (or only very loose gun stabilization) is a most promising arrangement.

Before presenting the evaluation of the alternative systems of stabilization, it is considered desirable to investigate the sources of error and the magnitudes of each.

[7] For explanation of this system see Section IV, "The Three-Switch Firing Proposal."

II. ERRORS OF MOVING TANK FIRE

Components of the Total Firing Error

All of the errors[8] in moving tank fire are included among the following:

1. "Inherent" errors which apply to the simplest situation of a tank standing still and shooting at a stationary target at known range. Some of the sources of this error include those due to inaccurate correction for barrel wear, nonstandard ballistic conditions, cross-winds, ammunition dispersion, fire control errors due to design limitations (such as backlash, parallax, superelevation input data, etc.), failure to sight exactly on target center, etc. This type of error will be called stationary fire error and will be denoted by the symbol A.

2. The error due to range estimation (whether range estimation is visual or with the aid of a range finder). This type of error will be called range error and will be denoted by the symbol R.

3. The error due to the movement of the gun. This type of error comes about in moving fire because the gun is constantly going through perturbations induced by the movement of the tank. The stabilization system reduces the amount of gun perturbations, but they still occur with sufficient angular travel to remain an important source of firing error. This type of error will be referred to as gun error and will be denoted by G.

4. The error due to sighting which comes about because of perturbations induced by the moving tank on the sight. The sight is assumed to be going through perturbations around some fixed line in space which is controlled by the gunner when he precesses the gyro by moving his handwheels. It is the departure of this sight line from a line to target center (at the time the gunner fires) which will be referred to here as the sight error and will be denoted by S.

Total Firing Error as a Function of Component Errors

Consider the case of stabilization in which the sight is slaved to the gun. Both the sight and the gun are instantaneously going through identical perturbations. There is a time lag, however, between the instant (a) that the gunner decides he is on target and wishes to fire, and the instant (b) in which the gun fires. This time lag is sufficiently long (approximately ½ second) so that there is no relationship between the gun positions at (a) and (b).[9] This means, of course, that choosing an instant to fire which is based on the sight movements is equivalent to picking a gun position at random from a distribution of gun positions.

Now consider the effect of each error in azimuth on the position of the shot. The gun and sight will be displaced a certain number of feet (S) from target center owing to aiming or sighting error—say position 1. This error is the distance between the line of sight and the line to target center at the time the gunner decides to fire. During the time lag, however, the gun has departed from position 1 a distance G to position 2. Its position is then $(G + S)$ feet from target center. The shell will depart from position 2

[8] With the exception of cant, lead estimation, and bending of the gun tube. These items were not considered because it was desired to reduce the calculations to manageable proportions. It is expected that none of the conclusions of this report will be affected by exclusion of these items from the study.

[9] AMRL, op. cit., Ref. 2.

Analysis of perturbation data for runs of the Centurion II tank also show that for a time lag of this type as large as ⅒ second there exists no relationship between the position of the gun at one instant and another.

during its flight a distance A to position 3, which is equal to $(G + S + A)$ feet from target center.[10] A similar argument holds for elevation.

In symbols, if T is the total error, then $T_A = S_A + G_A + A_A$ and $T_E = S_E + G_E + A_E + R_E$, where the subscripts A and E denote azimuth and elevation, respectively. Since both total errors are linear functions of random variables, the mean ($\overline{T}$) and the standard deviation (σ_T) of the distribution of total errors is given by

$$\overline{T}_A = \overline{S}_A + \overline{G}_A + \overline{A}_A \qquad \overline{T}_E = \overline{S}_E + \overline{G}_E + \overline{A}_E + \overline{R}_E \tag{1}$$

$$\sigma_{T_A}^2 = \sigma_{S_A}^2 + \sigma_{G_A}^2 + \sigma_{A_A}^2 \qquad \sigma_{T_E}^2 = \sigma_{S_E}^2 + \sigma_{G_E}^2 + \sigma_{A_E}^2 + \sigma_{R_E}^2 \tag{2}$$

It is expected that the form of the distribution of T_E and T_A will be approximately normal, since it is experimentally found that A is normally distributed while S, G, and R are reasonably close to being normally distributed, and the errors S, G, A, and R are not correlated with each other.[11]

Magnitudes of the Sources of Error

Stationary Firing Error

The value of σ_A will depend on the ammunition, gun, atmospheric conditions, etc., and is expected to be relatively stable at a value of approximately 0.2 mil.[12]

Range Error

The value of σ_R will depend on whether range estimation is visual or with the aid of a range finder. We have:

$$\sigma_R = \left[16.1\left(\frac{r}{\overline{V}}\right)^2\right]\sigma_{Est} \tag{3}$$

where r = true range

$\overline{V}$ = average shell velocity

σ_{Est} = standard deviation of range estimation error (in percent).

The derivation of equation 3 is given at the end of this section.

In the case of visual range estimation, $\sigma_{Est} = 1.25 \times$ (% Mean Range Estimation Error) where experimental data places the figure for MREE at from 17% to 20%.[13] The distribution of range errors is experimentally found to be normally distributed with a mean of zero.[14]

[10] The time sequence above is adopted for convenience of exposition. This constrains a difference in definition of the errors in which some errors defined under A are assumed to occur under G. Since addition is associative, however, the end result is not affected. It is to be expected that the errors S and G would be slightly correlated, but the influence of this is small and will be neglected.

[11] The distribution of shots from a moving tank has been found experimentally to be approximately normally distributed. See, for example, Figure 16 of AMRL Report, op. cit., Ref. 2.

[12] AMRL Report, op. cit., Ref. 2.
Also, F. I. Hill, BRL Report No. 739, "Report of First Tank Conference Held at Aberdeen," SECRET. (Ref. 3).

[13] Various other studies of visual range estimation at the Artillery School, Fort Sill, Oklahoma, and the Armored School, Fort Knox, Kentucky, place this figure at from 25% to 50%. For the purpose of this report, however, the 17% to 20% figure is considered the more reliable (BRL Report No. 739, above).

[14] Hill, Peterson, and Zeller, BRL Report No. M554, "A Study of the Range Finder for the Light Tank T41E1," (Ref. 4). Also Ref. 3.

In the case of a coincidence range finder on a moving tank, Aberdeen tests indicate that the mean range estimation error is about 7%.[15] (Since no data was available at the time of this report on the accuracy of the stereoscopic or stadiometric range finder in stabilized fire, only the coincidence range finder is considered.)

The calculations in this report are based on a mean range estimation error of 5% and 10% to represent upper and lower limits of accuracy of stabilized fire with a range finder. Calculations based on a mean range estimation error of 17% and 20% represent the upper and lower limits of accuracy when visual estimation is used.

The range error is also a function of the kind of ammunition used and the caliber of the gun. Calculations for the 75 mm and 90 mm gun using APC and HVAP shot are included in this report.

Gun Error

The magnitude of σ_G will depend on the tightness of stabilization, the suspension system of the tank, the weight of the gun, the character of the terrain over which the tank is traveling, the speed of the tank, etc. This figure has been obtained for various combinations of the above conditions. A summary of this data, which is given in Table 3, is derived from boresight camera records of gun movements during tank runs under test conditions.

On some tests the character of the ground was clearly not representative of battle ground, and for most tests the character of the ground was unknown. Much better data of this type is needed for a more refined analysis.

Sight Error

The gunner acts somewhat as an averaging mechanism in determining the sight line; that is, he attempts to place his average gun position (and hence sight position) on the target. It appears obvious that the larger the perturbations of the gun, the more the error which will be made in this attempt. No quantitative data on the magnitude of this error, however, has been found.

It is the opinion of the writer that a figure of $\sigma_S = 3/4$ mil represents a minimum figure for present-day equipment with sight and gun stabilized together.

In the opinion of design engineers, $\sigma_S = 1/4$ mil is a reasonable goal for the system wherein the sight and gun are separately stabilized. It is expected to be so small because of the much reduced perturbations of the sight when it is stabilized separately.

Needless to say, the desirability of separate stabilization of gun and sight, as compared with combined stabilization, depends exclusively on the data used for σ_S.

[15] Report of the First Tank Conference held at Aberdeen, op. cit., Ref. 3:

"The Tank T41, equipped with a coincidence type range finder, was operated over smooth, medium, and severe terrain. The operator was instructed to range on a fixed target as frequently as possible. Three operators were used to obtain 363 moving observations at ranges between 500 and 2500 yards, with the result that only 5.2% of the observations were in error by more than 17% of true range." On the assumption that the distribution of the 363 observations (for percent of true range) is normally distributed with a mean of zero, it follows from normal curve considerations that MREE = 7%. Because of the large spread of true ranges, the assumption of a normal distribution is a bit weak. However, since it is found experimentally that, for a fixed range, this value is about normally distributed, it is felt that a MREE of 7% as calculated is satisfactory for a rough figure of the accuracy of ranging with a range finder in stabilized fire.

Derivation of Expression for Range Error Standard Deviation

To derive: $\sigma_R = 16.1\left(\frac{r}{\overline{V}}\right)^2 \sigma_{Est}$ (16)

where: σ_R = standard deviation of shot due to range estimation error
r = true range
$\overline{V}$ = average velocity of shell
σ_{Est} = standard deviation of range estimation error (in %).

Let Δh be the height of the projectile above the line of sight as it flies along its trajectory, and let the subscript e refer to an estimated condition. At the estimated range, r_e,

$$\Delta h = 0 = r_e \tan\theta - \tfrac{1}{2} g t_e^2$$

where θ is the angle of elevation of the gun
t_e is the time of flight of the projectile (including drag)
g is the acceleration of gravity.

Then $t_e = \dfrac{r_e}{\overline{v}_e} \approx \dfrac{r}{\overline{V}}$

where $\overline{V}$ is the *mean* speed of the projectile to the range considered.

From the above two equations we have

$$\tan\theta = g\frac{r_e}{2\overline{v}_e^2}, \text{ since } r_e \tan\theta = \tfrac{1}{2} g t_e^2 = \tfrac{1}{2} g\left(\frac{r_e}{\overline{v}_e}\right)^2$$

However, at the true range to the target, r,

$$\Delta h = r\tan\theta - \tfrac{1}{2}g\left(\frac{r}{\overline{V}}\right)^2$$
$$= g\frac{rr_e}{2\overline{v}_e^2} - \tfrac{1}{2}g\left(\frac{r}{\overline{V}}\right)^2.$$

If the range estimation error is a constant fraction, p, of the true range, then

$$r_e = (1+p)\,r$$

Also $\overline{v}_e = \overline{V}$

and $$\Delta h = (1+p)\frac{g}{2}\left(\frac{r}{\overline{V}}\right)^2 - \frac{g}{2}\left(\frac{r}{\overline{V}}\right)^2$$
$$= \frac{1}{2}pg\frac{r^2}{\overline{V}^2}$$
$$\sigma_R = 16.1\left(\frac{r}{\overline{V}}\right)^2\sigma_{Est}$$

[16] Derivation supplied by Aberdeen Proving Ground and appears in Hill et al, op. cit., Ref. 4.

TABLE 3.

Summary of Results on Continuous Tracking—Test of 1948

Chart No.	Duration of Run	Course	Speed of Tank mph	Gunner	CENTURION Horizontal S. D. in mils	CENTURION Vertical S. D. in mils
I	1′ 42″	Straight	6	A	0.8	1.0
II	1′ 46″	Straight	6	A	0.6	0.9
III	1′ 47″	Straight	6	B	1.0	1.4
IV	1′ 53″	Straight	6	B	0.6	0.9
V	1′ 48″	Straight	6	C	0.9	1.6
VI	42″	Straight	15	A	1.0	1.6
VII	45″	Straight	15	B	0.9	1.4
VIII	45″	Straight	15	B	1.1	1.5
IX	44″	Straight	15	C	1.2	1.6
X	1′ 58″	Zigzag	6	B	2.0	1.3
XI	1′ 50″	Zigzag	6	C	2.3	1.7
XII	2′ 10″	Zigzag	6	A	1.8	0.9
XIII	49″	Zigzag	15	A	2.6	1.8
XIV	48″	Zigzag	15	A	3.0	2.0
XV	44″	Zigzag	15	B	4.1	2.2
XVI	48″	Zigzag	15	C	3.5	2.1
XVII	1′ 0″	Circular	6	C	4.3	1.8
XVIII	1′ 0″	Circular	6	C	3.4	1.8
XIX	1′ 0″	Circular	6	B	2.6	1.3
XX	1′ 0″	Circular	6	B	3.3	1.4
XXI	1′ 0″	Circular	6	A	3.7	1.8
XXII	1′ 0″	Circular	6	A	2.9	2.1
					M4 (STANDARD)	
XXIII	1′ 42″	Straight	6	C	8.8	2.2
XXIV	1′ 33″	Straight	6	A	6.3	1.8
XXV	1′ 41″	Straight	6	B	5.2	2.0
XXVI	47″	Straight	15	C	12.0	2.7
XXVII	45″	Straight	15	A	10.3	4.0
XXVIII	46″	Straight	15	B	7.0	3.1
XXIX	1′ 56″	Zigzag	6	C	27.0	5.2
XXX	1′ 52″	Zigzag	6	A	18.9	3.1
XXXI	1′ 50″	Zigzag	6	B	27.0	2.9

52. Data provided by Aberdeen Proving Ground.

III. SEPARATE STABILIZATION OF THE SIGHT AND GUN

Because of the large contribution of the sight error and the gun error to the total error (particularly for moderate ranges at which tank battles with stabilized tanks would take place), it is desirable to reduce one or both of these errors. This may be accomplished for the sight by separate and tighter stabilization and for the gun by either tighter stabilization or by a firing mechanism which will not permit the gun to fire when the gun error is too large. This section is concerned with the possibilities of tighter stabilization.

The gun, being a rather large mass, requires a considerable amount of power to accomplish a tighter stabilization. Since no additional large amounts of power can be made available without unduly increasing the size, cost, and complexity of the stabilization equipment, tighter stabilization of the gun is generally considered not feasible. It has, however, been thought that the sight may be more tightly stabilized either by making small additional amounts of power available for the total stabilization function or by using some of the power presently used to stabilize the gun. It is then desired to determine the effect on total error (and, hence, single-shot probability of hit) which would result from such a separate stabilization of gun and sight.[17]

Figures 1 to 24 show probability of hitting a 7½ ft. square target (as a function of range) for various values of gun error, sight error, and factors affecting the range error.[18]

With the assumption that the current value of σ_s is ¾ mil and that the additional power to be made available would enable a reduction of σ_s to ¼ mil, Table 4 gives for each of these sight errors the probability of hit for various values of gun error for two ammunition types and for visual and optical ranging.

Several conclusions may be drawn from Table 4:

(1) The greatest increases in probability, when the sight error is decreased, take place at smaller ranges—in general, at ranges less than 1000 yds.

(2) The greatest increases in probability, when the sight error is decreased, occur when the gun error is small: in general, the gun error has marked influence in overshadowing an improvement in sight error when the gun error is greater than 1½ mils for ranges of less than 700 yds. and 1 mil for ranges up to 1500 yds.

(3) For almost all conditions shown in the table, the gain in probability of hit due to lowering the sight error is approximately the same whether AP or HVAP shot is used.

In general, the table merely states that a moderate decrease in one component of the total error will yield significant increases in the probability of hit, provided that the other components are not too large in relation to it. This is, of course, a well-known principle. The primary value of Table 4 is that it gives a quantitative meaning to this statement. Table 4 indicates that it may be worthwhile to decrease the sight error, provided that no increase in the gun error takes place.[19]

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[17] See also Section V of this report dealing with stabilization of the sight only.

[18] When interpreting the data on probability of hit, it is necessary to keep in mind that some kinds of errors have not been included, viz. lead estimation, cant correction, and bending of the gun tube. Thus, these figures are to be compared only for relative values under different conditions and are not expected to agree with field trials.

[19] If other factors are equal, the probability of hit will remain unchanged even though σ_S and σ_G are changed, provided that $(\sigma_S^2 + \sigma_G^2)$ is kept constant.

TABLE 4.

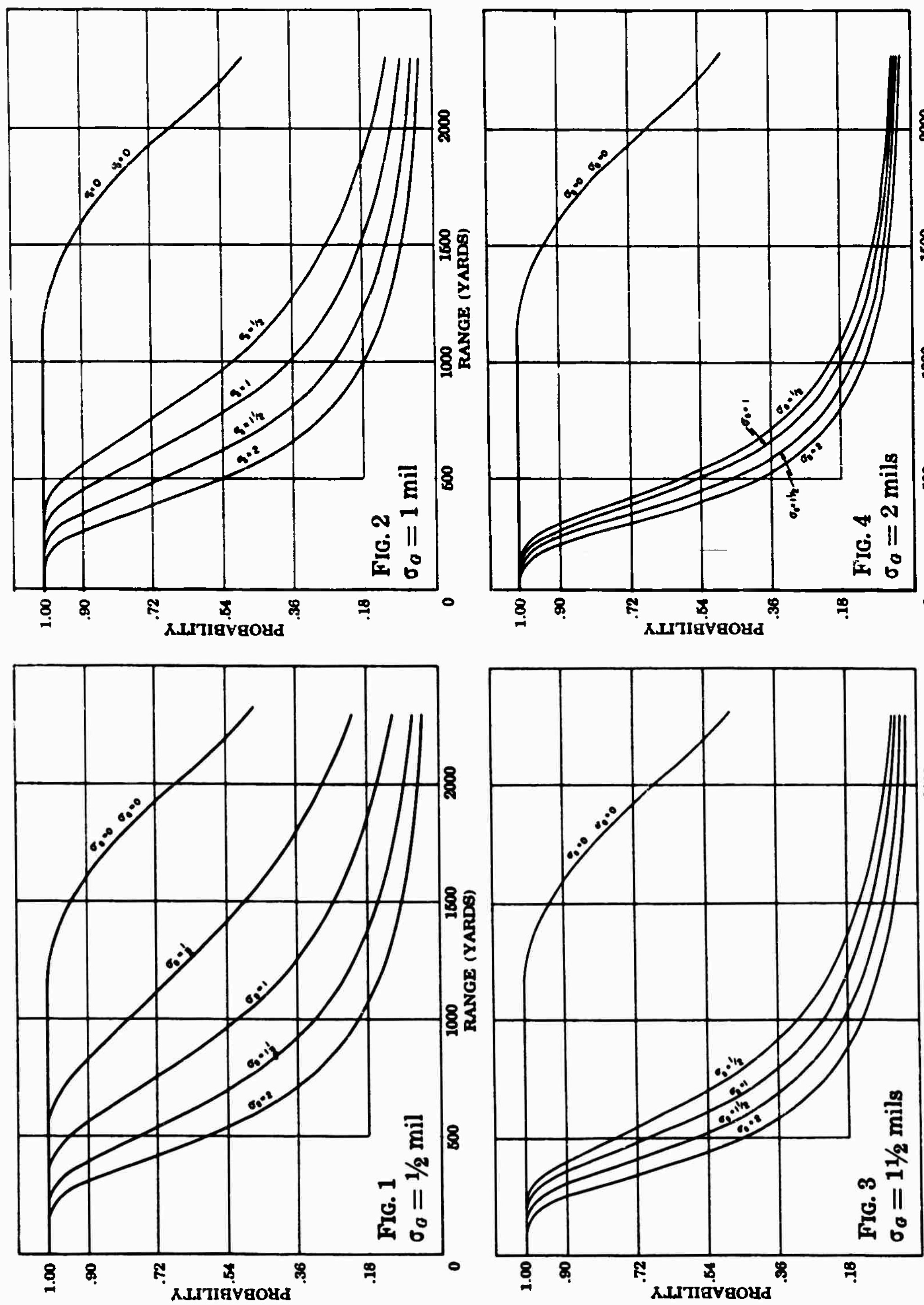
Increase in Probability of Hitting a 7½ ft. Square Target due to Decreasing the Sighting Error from ¾ mil to ¼ mil.

AP Shot

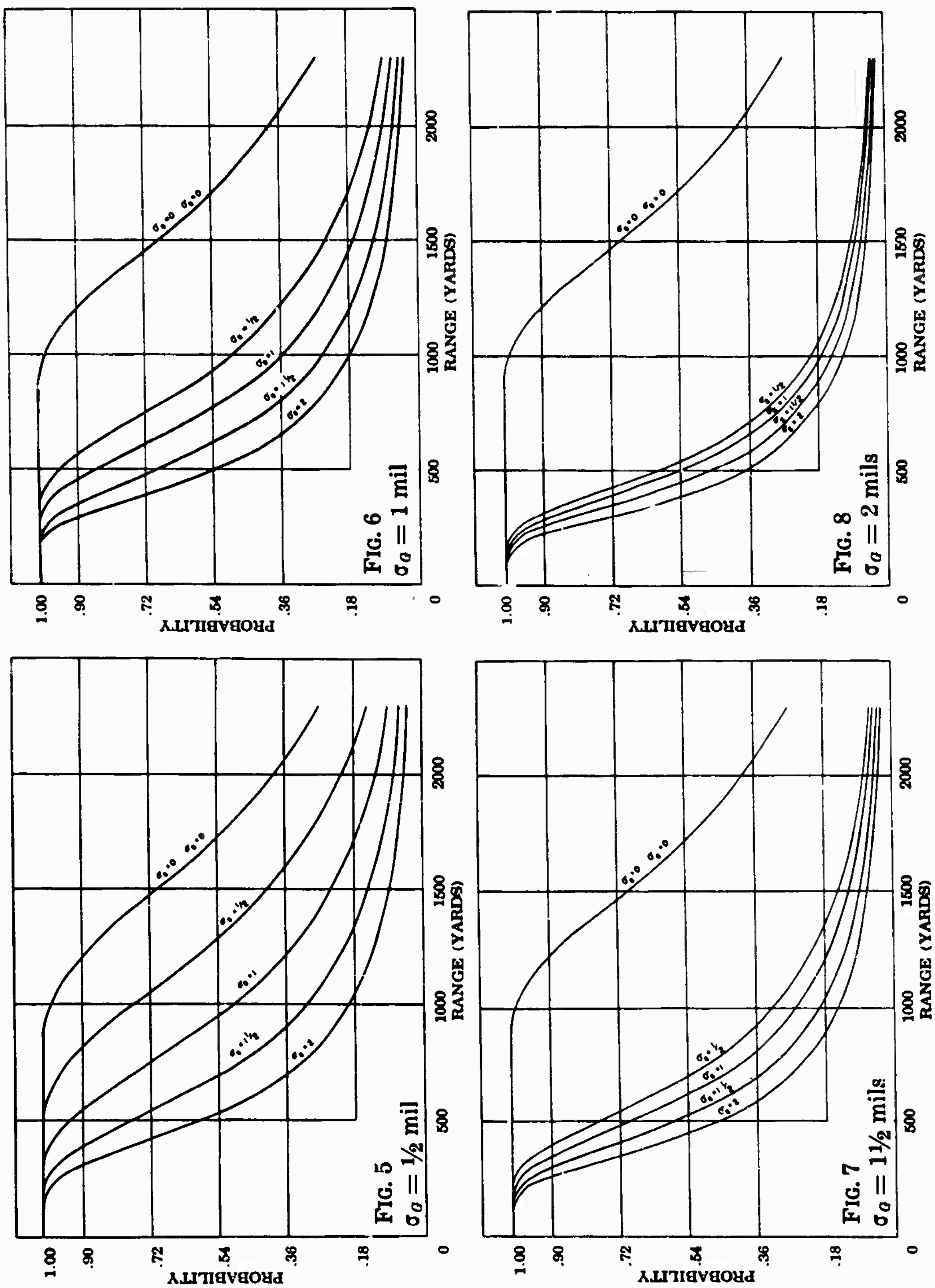
		MREE = .10						MREE = .20					
	G(mil)	½		1		1½		½		1		1½	
Range	*S*(mil)	¾	¼	¾	¼	¾	¼	¾	¼	¾	¼	¾	¼
300		1.00	1.00	1.00	1.00	.97	.99	1.00	1.00	1.00	1.00	.96	.98
500		.99	1.00	.88	.99	.73	.81	.94	1.00	.85	.95	.71	.79
700		.85	.98	.68	.82	.49	.58	.75	.88	.60	.73	.46	.53
1000		.57	.80	.40	.51	.26	.32	.38	.54	.31	.39	.22	.26
1300		.36	.60	.24	.32	.16	.20	.23	.31	.17	.21	.12	.14
1500		.26	.41	.13	.23	.11	.14	.16	.22	.12	.14	.09	.10
1700		.20	.32	.13	.17	.09	.11	.11	.15	.08	.10	.06	.07
2000		.13	.21	.08	.12	.06	.08	.07	.10	.05	.07	.04	.05

HVAP Shot

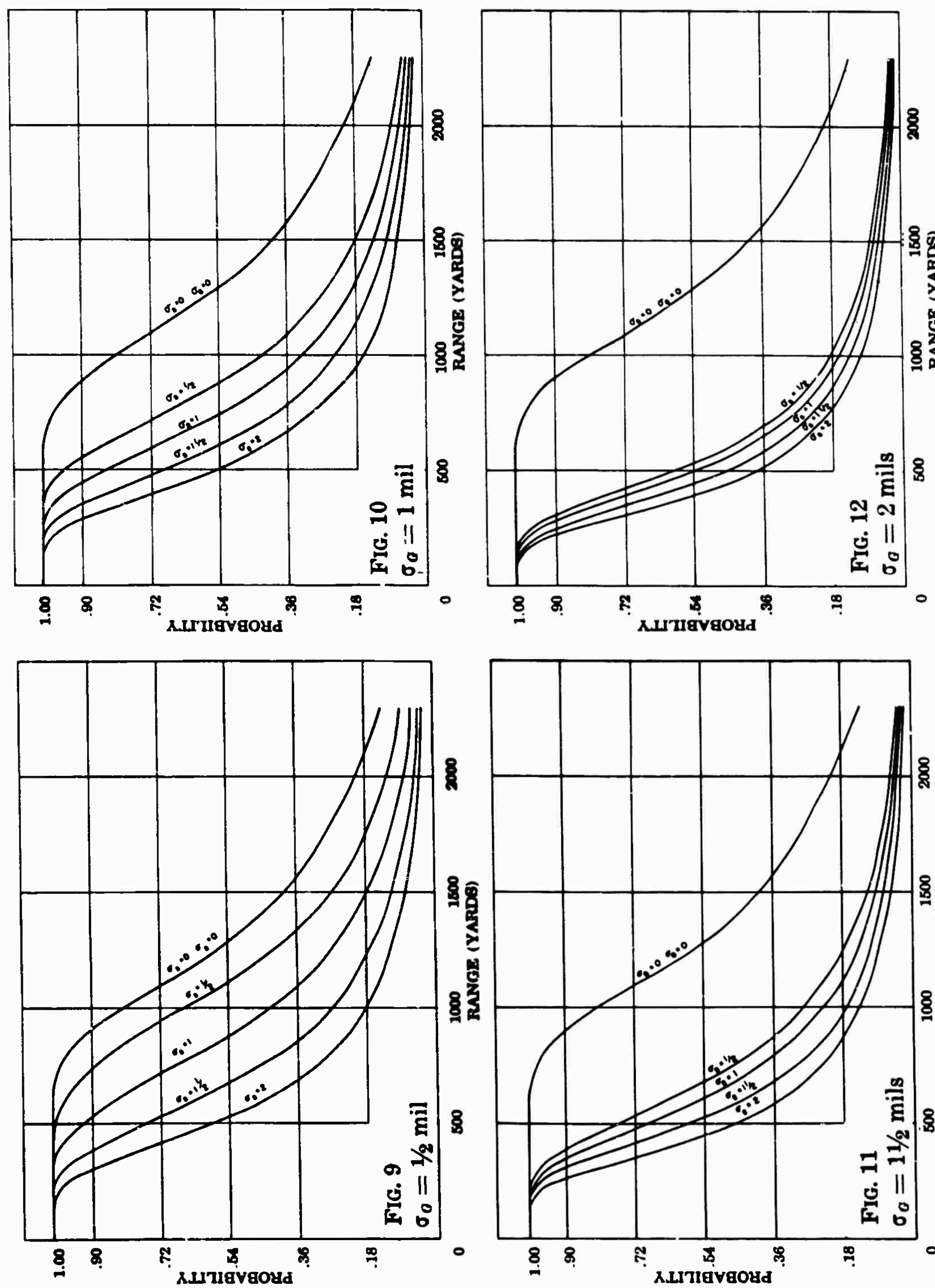
300		1.00	1.00	1.00	1.00	.97	.99	1.00	1.00	1.00	1.00	.96	.99
500		.99	1.00	.89	.99	.73	.81	.96	1.00	.89	.99	.72	.80
700		.91	1.00	.69	.84	.49	.59	.78	.97	.66	.80	.47	.55
1000		.63	.89	.42	.55	.29	.33	.49	.73	.37	.47	.26	.31
1300		.43	.64	.27	.36	.17	.21	.32	.45	.22	.29	.15	.19
1500		.32	.49	.21	.28	.13	.16	.23	.32	.16	.21	.11	.13
1700		.25	.39	.16	.21	.10	.12	.16	.22	.12	.15	.08	.09
2000		.17	.27	.11	.13	.07	.08	.10	.14	.07	.09	.05	.07



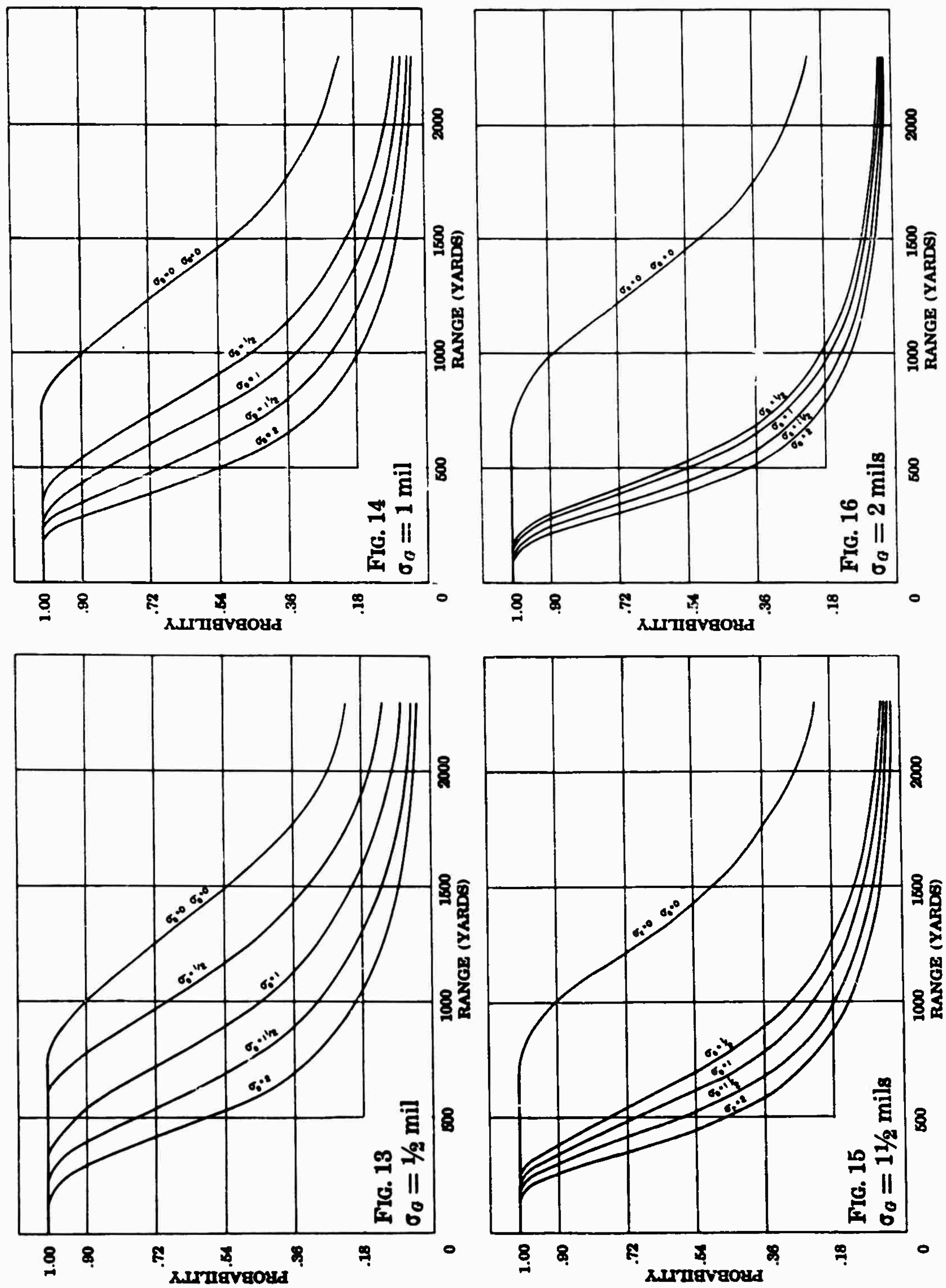
FIGS. 1 to 4. Probability of Hitting a 7½ ft. Square Stationary Target for Stabilized Fire HVAP shot, MREE = .05



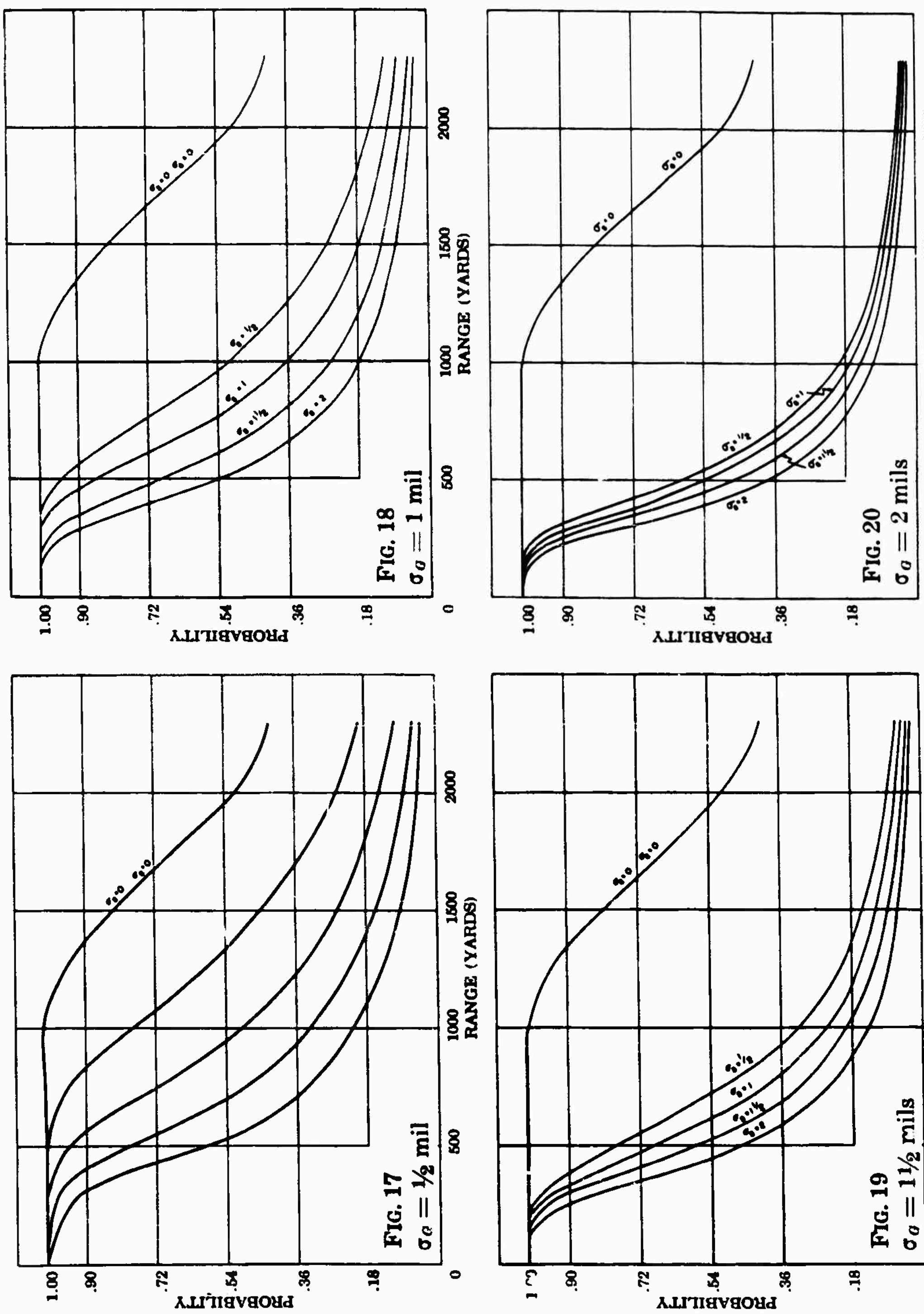
FIGS. 5 to 8. *Probability of Hitting a 7½ ft. Square Stationary Target for Stabilized Fire HVAP shot, MREE = .10*



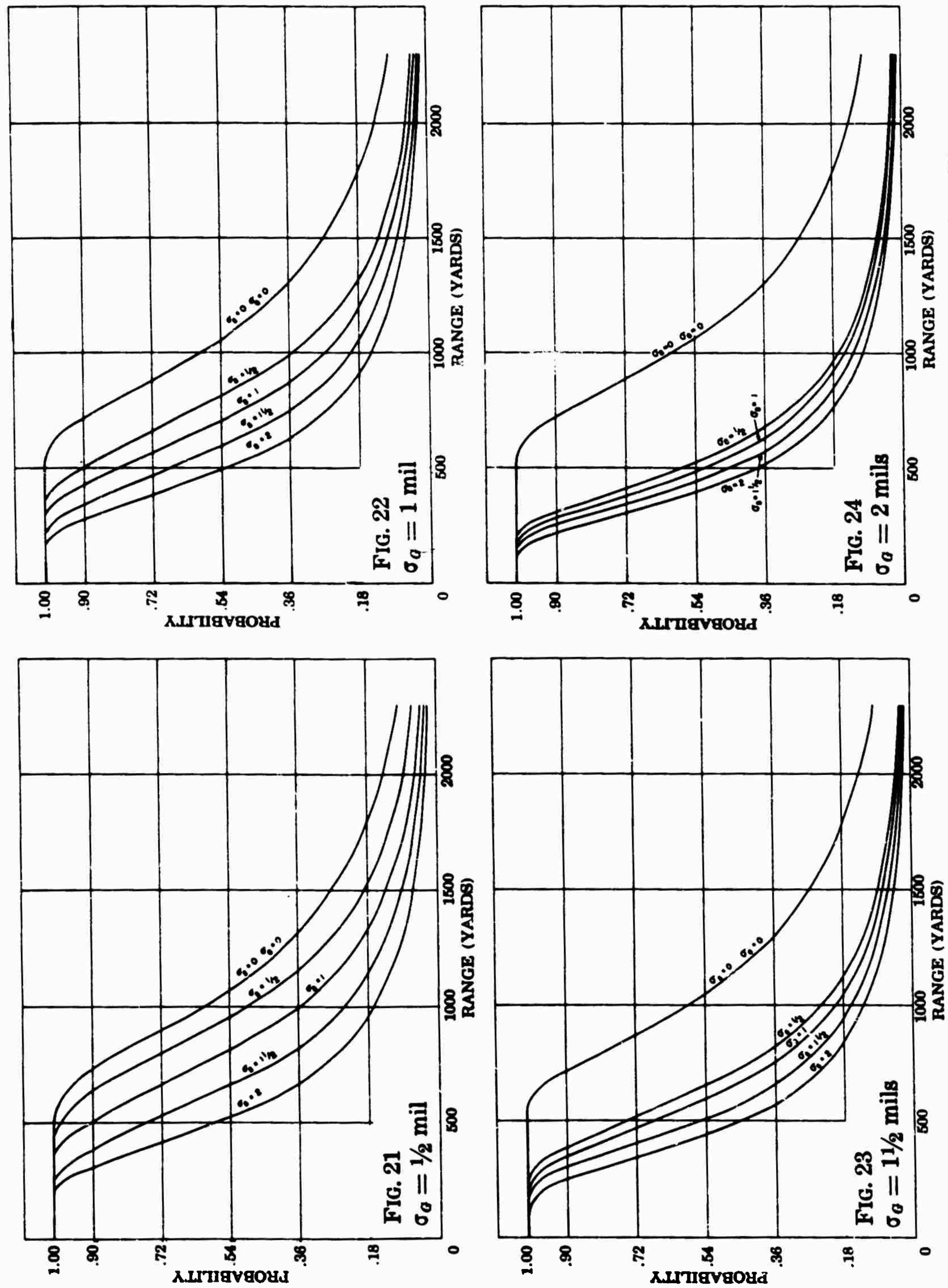
FIGS. 9 to 12. *Probability of Hitting a 7½ ft. Square Stationary Target for Stabilized Fire HVAP shot, MREE = .20*



FIGS. 13 to 16. *Probability of Hitting a 7½ ft. Square Stationary Target for Stabilized Fire AP shot, MREE = .05*



FIGS. 17 to 20. *Probability of Hitting a 7½ ft. Square Stationary Target for Stabilized Fire AP shot, MREE = .10*



FIGS. 21 to 24. *Probability of Hitting a 7½ ft. Square Stationary Target for Stabilized Fire AP shot, MREE = .20*

IV. THE THREE-SWITCH FIRING PROPOSAL

As previously stated, the probability of a hit may be increased by reducing the gun error. This may be accomplished by tighter stabilization of the gun, but the fact that much additional power is required to accomplish the aim in this manner precludes any substantial gain in over-all accuracy by this method.[20]

An alternative procedure of reducing the gun error by a device referred to as the three-switch proposal has been suggested.[21] The essence of this proposal is that the gun is permitted to fire only when it is within a certain distance (both in azimuth and elevation) from the sight gyro line. Mechanically, the gunner operates the first switch when he wishes to fire; if the gun is within a distance $\pm A$ in azimuth from the line of the sight gyro and also within a distance $\pm E$ in elevation from the line of the sight gyro, then it will fire. If the gun is not within this region (called the firing region), then it will not fire until, as a result of its perturbations, it moves into the region.

This device effectively results in a refusal to fire unless at least a certain probability of hit is assured; or conversely, if the shot is going to miss the target (with high probability), the gun will not be fired.[22]

Since, under the three-switch proposal, firing will sometimes be delayed, it is necessary to consider the probability of a delay in firing and the probable duration of a delay, as well as the accuracy of firing when it takes place. In addition to previously mentioned factors affecting accuracy, the size of the firing region must be considered. As the firing region is made smaller, the accuracy of fire imposed by the three-switch method increases. So also, unfortunately, does the frequency of delays in firing and their probable duration.

Accuracy of Fire

To obtain the probability of hit with the three-switch arrangement, the following quantities are needed: (1) the probability of a delay in firing (P_D); (2) the probability of hit in the case of instant firing (P_I); and (3) the probability of hit in the event of a delay in firing (P_F). This last probability has been calculated by a method known as "monte carlo."[23] The probability of hit calculations of this section are approximations due to the simplifying assumption covered in the next section as well as the factors enumerated in this section.

Let P^* be the total probability of hit. Then

$$P^* = (1.- P_D)\, P_I + P_D \cdot P_F \tag{4}$$

[20] It will be seen in this section that a greater gain is made by the three-switch proposal than by maintaining the present degree of gun stabilization when σ_G is large, as in the case of high speed on rough ground.

[21] This general type of device is not new. It was used during World War II by the Germans and during the Korean conflict on Russian-built tanks. A similar device is also used in aircraft fire control.

[22] It may also be desirable to limit the ability of the gun to fire to instances where the angular velocity is less than some quantity to be determined by the size of the firing region. See pp 37.

[23] For information on the nature of this type of calculation procedure see:

(a) G. W. King, "Operational Analysis," *Proceedings of the Annual Middle Atlantic Conference*, February 1951, Ref. 5.

(b) Proceedings of the following seminars by International Business Machines Corporation: Scientific Computation (Nov. 1949), Computation (Dec. 1949) and Industrial Computation (Sept. 1950). (Ref. 6).

(c) "Stochastic (Monte Carlo) Attenuation Analysis," Rand Corporation, 1949. (Ref. 7).

where P_D depends on the dimensions of the firing region and the gun perturbations as follows for azimuth:

$$(1 - P_D) = \frac{1}{\sigma_{G_A}\sqrt{2\pi}} \int_{-A}^{A} exp\left[- \frac{1}{2} \left(\frac{x}{\sigma_{G_A}} \right)^2 \right] dx \qquad (5)$$

and similarly for elevation.

$A = \frac{1}{2}$ the side of a firing region and σ_G^2 is the variance of gun perturbations taken along the time scale (the gun perturbations assumed to be normally distributed). The value of P_I is obtained from

$$P_I = \frac{1}{\sigma_T\sqrt{2\pi}} \int_{-3.75}^{3.75} exp\left[- \frac{1}{2} \left(\frac{x}{\sigma_T} \right)^2 \right] dx \qquad (6)$$

where 3.75 is one-half the side of a $7\frac{1}{2}$ ft. square target and $\sigma_T^2 = \sigma_A^2 + \sigma_R^2 + \sigma_S^2 + \mu_G$.

$$\mu_G = \sigma_G^2 \left[1 - \frac{\frac{1}{\sqrt{2\pi}} \left(\frac{A}{\sigma_G} \right) exp\left[- \frac{1}{2} \left(\frac{A}{\sigma_G} \right)^2 \right]}{\frac{1}{\sigma_G\sqrt{2\pi}} \int_0^A exp\left[- \frac{1}{2} \left(\frac{x}{\sigma_G} \right)^2 \right] dx} \right] \qquad (7)$$

where A and σ_G have the same meaning as previously defined.[24]

[24] By definition, the variance (square of the standard deviation) of any distribution is: $\mu_2 = \dfrac{\int x^2 f(x)dx}{\int f(x)dx}$

wherein for a normal curve $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}} exp\left(\dfrac{-x^2}{2\sigma^2}\right)$

and the limits are the truncating points $+A$ and $-A$. After substitution, integrating by parts and simplifying

$$\mu_2 = \frac{\frac{1}{\sigma\sqrt{2\pi}} \int_{-A}^{A} x^2\, exp\left(\frac{-x^2}{2\sigma^2}\right) dx}{\frac{1}{\sigma\sqrt{2\pi}} \int_{-A}^{A} exp\left(\frac{-x^2}{2\sigma^2}\right) dx} = \sigma^2 \left[1 - \frac{\frac{1}{\sqrt{2\pi}} \left(\frac{A}{\sigma} \right) exp\left[- \frac{1}{2} \left(\frac{A}{\sigma} \right)^2 \right]}{\frac{1}{\sigma\sqrt{2\pi}} \int_0^A exp\left(\frac{-x^2}{2\sigma^2}\right) dx} \right]$$

A graph of this function is shown on Figure 25.

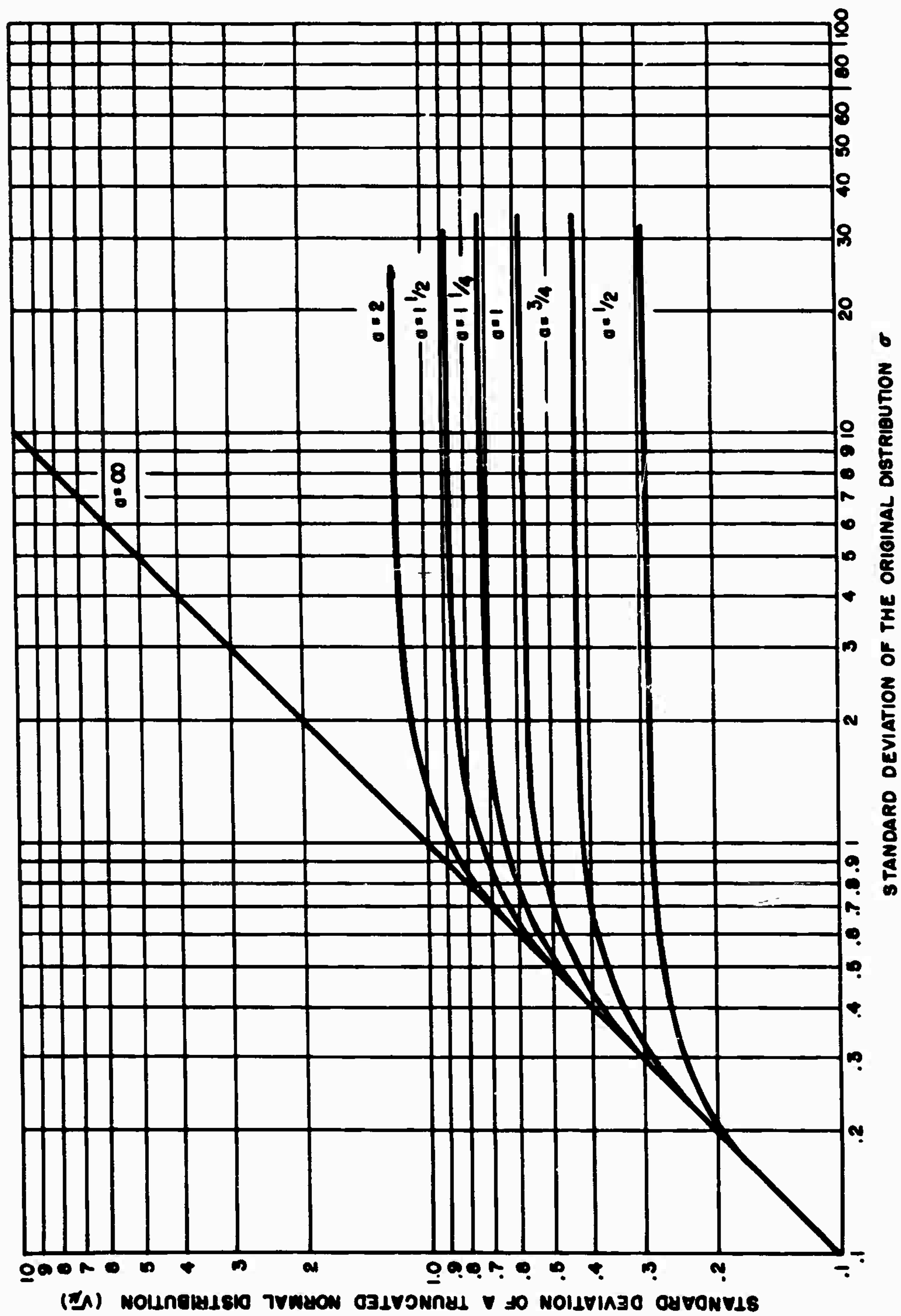


FIGURE 25. *Standard Deviation,* $(\sqrt{u})$*, of a Truncated Normal Distribution as a Function of the Standard Deviation of the Original Distribution and the Truncating Points* $(\pm A)$.

The values of the quantities u_G and $(1 - P_D)$ for various values of σ_G and A are shown in Table 5. The data in this table is based on a normal distribution of gun movements.

TABLE 5.
Variance (μ_G) of a Truncated Normal Distribution with Original Variance (σ_G) and Truncating Points ($\pm$A).

		Variance of Gun Perturbations under Firing Region Restrictions u_G				Probability of Instant Firing (1-P_D)			
σ_G	A	½	1	1½	2	½	1	1½	2
1		.081	.292	.549	.775	.3829	.6827	.8666	.9545
1½		.082	.299	.653	.968	.2358	.4952	.6827	.8175
2		.084	.326	.780	1.166	.1974	.3829	.5467	.6827

(Normal distribution of gun movements assumed.)

Table 6 shows the probability of hitting a 7½ ft. square target for various values of the different components of the total error and for various dimensions of the firing region. This data has been calculated under the following assumptions (some of which are made in order to reduce the calculations to manageable proportions):

TABLE 6.
Probability of Hitting a 7½ ft. Square Target for Various Dimensions of the Firing Region and for Errors of Varying Magnitudes.*

Range (yds.)	A= E= (mils) ½	1	1½	2	∞
	σ_S = ½ mil		σ_G = 1 mil		
400	.988	.979	.967	.959	.950
700	.621	.576	.542	.501	.500
1000	.323	.268	.240	.229	.225
	σ_S = ½ mil		σ_G = 1½ mil		
400	.988	.976	.954	.931	.848
700	.621	.569	.495	.429	.382
1000	.323	.260	.206	.189	.166
	σ_S = ½ mil		σ_G = 2 mil		
400	.987	.975	.946	.921	.713
700	.621	.567	.475	.378	.286
1000	.323	.257	.188	.159	.124
	σ_S = ¾ mil		σ_G = 1 mil		
400	.979	.962	.948	.938	.931
700	.588	.539	.493	.471	.458
1000	.293	.240	.217	.209	.206

TABLE 6—Continued

Range (yds.)	A= E= (mils) ½	1	1½	2	∞
	σ_S = ¾ mil		σ_G = 1½ mil		
400	.980	.958	.928	.906	.828
700	.587	.535	.459	.405	.380
1000	.293	.236	.188	.174	.156
	σ_S = ¾ mil		σ_G = 2 mil		
400	.978	.956	.919	.883	.690
700	.586	.532	.440	.360	.273
1000	.293	.235	.173	.150	.119
	σ_S = 1 mil		σ_G = 1 mil		
400	.958	.937	.918	.904	.895
700	.525	.482	.444	.427	.423
1000	.219	.197	.194	.187	.186
	σ_S = 1 mil		σ_G = 1½ mil		
400	.958	.932	.897	.862	.790
700	.525	.477	.407	.367	.334
1000	.218	.186	.166	.154	.143
	σ_S = 1 mil		σ_G = 2 mil		
400	.958	.930	.886	.841	.662
700	.525	.473	.390	.333	.257
1000	.218	.181	.154	.134	.113
	σ_S = 1¼ mil		σ_G = 1 mil		
400	.921	.896	.873	.855	.850
700	.454	.425	.396	.383	.387
1000	.215	.181	.170	.166	.163
	σ_S = 1¼ mil		σ_G = 1½ mil		
400	.921	.891	.851	.869	.745
700	.453	.419	.368	.334	.305
1000	.211	.178	.154	.142	.132
	σ_S = 1¼ mil		σ_G = 2 mil		
400	.921	.889	.839	.775	.631
700	.453	.416	.352	.304	.240
1000	.215	.176	.146	.126	.105

* In these calculations the stationary firing error was taken to be 0.2 mil standard deviation, and the mean range estimation error was taken to be 17%. The component probability of hit in the event of delayed firing (P_F) was computed by monte carlo calculations based on 1000 trials in which all variables were assumed normally distributed with a mean of zero.

(1) That the resultant error is normally distributed.[25]

(2) That the variance of the distribution of sight errors for azimuth is equal to that of elevation, and similarly for the distribution of errors due to gun movements.[26]

(3) That the distribution of gun movements is normal.[27]

(4) That, in the event of delayed firing, the gun is equally likely to enter the firing region at any point along the boundary of that region.[28]

(5) That the firing region is assumed to be a square.[29]

One notes the rather obvious fact that very substantial gains in accuracy are to be expected through use of the three-switch proposal. This is particularly true for the intermediate ranges at which stabilized tanks will fight tank battles, and at smaller ranges when the gun error is large (as in the case of high speed, rough ground, and/or evasive maneuver by the stabilized tank).

The latter conclusion then suggests the possibility of the three-switch proposal acting as a substitute for gun stabilization. This alternative is discussed under the section on stabilizing the sight only. In addition, the alternative of using the three-switch proposal in conjunction with gun stabilization (to a loose and tight degree) is considered in the same section.

The results of monte carlo calculations for the probability of hit in the event of delayed firing is given in Table 7. Data in this table applies to different values of mean range estimation error and to the 75 mm and 90 mm gun firing APC and HVAP shot. As has been previously stated, the firing regions so far considered are not optimal. Some considerations in obtaining optimal dimensions are considered in this section, and further study is recommended.

Optimal Firing Region Dimensions

The first thing which suggests itself in connection with the firing region dimensions is the desirability of decreasing the elevation side of the region. The elevation error is larger, owing to the error of range estimation, and if the target being fired at is a square target and the total variance in elevation is larger than that in azimuth, a greater gain in probability of hit is obtained by reducing the elevation error than by reducing the azimuth error an equal amount.

[25] The justification for this assumption is that the components of the total error are found to be reasonably close to being normally distributed. It can be shown that the sum of random variables approaches a normal distribution as the number of variables increases. Experimentally it is found that the sum of as few as three random variables is close to being normally distributed, particularly where the components themselves are nearly so distributed.

[26] This assumption is justifiable on the basis of requiring only approximate probability results. Similar calculations could, of course, be made without this assumption.

[27] This assumption is justified on the basis of the approximateness of required results. Available data (from strip camera records) indicates that this assumption is not unsatisfactory for the purpose in which it was used.

[28] It is felt that this is the weakest assumption involved in these calculations. However, it is also felt that it is not so weak as to impair seriously the validity of the results.

[29] Since the firing region may be designed to be whatever shape seems desirable, this is not an assumption which invalidates the results but rather one which restricts the results, so to speak, to one situation. It will be shown later in this section that this assumption tends to understate the probability of hit under the three-switch proposal compared with the probability which would result from "optimum" dimensions of the firing region.

TABLE 7.

Probability of Hitting a 7½ ft. Square Target with Delayed Fire (P_F), Assuming a Firing Region of ± a Mils in Azimuth and Elevation.

	75 MM Gun A P Shot				90 MM Gun HVAP Shot*											
	17% MREE				20% MREE				10% MREE				5% MREE			
$\sigma_s{}^A$	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0	0.5	1.0	1.5	2.0
400 Yds																
0.5	.99	.97	.94	.89	1.00	1.00	.99	.97	1.00	.92	1.00	.98	1.00	1.00	.99	.98
0.75	.98	.95	.91	.85	1.00	.99	.95	.92	1.00	1.00	.98	.92	1.00	1.00	.98	.95
1.00	.96	.93	.87	.79	.99	.97	.94	.86	.98	.98	.94	.89	.98	.97	.94	.88
1.25	.92	.89	.83	.72	.96	.96	.87	.83	.97	.95	.90	.78	.96	.94	.90	.77
700 Yds																
0.5	.62	.56	.45	.27	.94	.84	.58	.27	.99	.91	.39	.24	.99	.92	.61	.23
0.75	.59	.53	.42	.28	.92	.76	.48	.27	.94	.82	.50	.27	.95	.83	.52	.25
1.00	.53	.47	.37	.27	.84	.64	.43	.29	.96	.69	.44	.27	.88	.70	.45	.26
1.25	.45	.41	.33	.25	.57	.57	.38	.27	.70	.58	.40	.27	.72	.60	.41	.25
1000 Yds																
0.5	.32	.25	.17	.11	.50	.45	.24	.09	.84	.52	.22	.05	.90	.56	.23	.03
0.75	.29	.23	.15	.11	.26	.15	.22	.11	.68	.45	.26	.12	.75	.46	.22	.08
1.00	.25	.20	.14	.11	.24	.14	.20	.15	.52	.38	.24	.14	.44	.38	.22	.12
1.25	.22	.17	.14	.10	.36	.30	.21	.15	.40	.31	.22	.15	.42	.31	.20	.12

* Based on 250 observations on a Monte Carlo calculation instead of 1000. Hence this data is subject to twice as much sampling fluctuation as is the 75 mm gun.

As previously asserted:

$$\sigma_{T_A}{}^2 = \sigma_{A_A}{}^2 + \sigma_{S_A}{}^2 + \sigma_{G_A}{}^2$$

and

$$\sigma_{T_E}{}^2 = \sigma_{A_E}{}^2 + \sigma_{S_E}{}^2 + \sigma_{G_E}{}^2 + \sigma_R{}^2$$

Now on the assumptions that $\sigma_{A_A} = \sigma_{A_E}$, $\sigma_{S_A} = \sigma_{S_E}$, $\sigma_{G_A} = \sigma_{G_E}$,[30] then $\sigma_{T_E} > \sigma_{T_A}$ and it is desired to reduce both to $\sigma_{T_E}' = \sigma_{T_A}'$ where $\sigma_{T_E}' < \sigma_{T_E}$ and $\sigma_{T_A}' < \sigma_{T_A}$. The reduction is to be accomplished by appropriate choice of the firing regions such that

$$\mu_E = \mu_A - \sigma_R{}^2 = \mu_A - \left[16.1\,\sigma_{\mathrm{Est}}\left(\frac{r}{\overline{V}}\right)^2\right]^2$$

[30] As shown by Table 3, this assumption is not true. It is made here only for clear demonstrating of the point, which is conceptually the same if the assumption is removed.

It is thus seen desirable to have a varying firing region for elevation which is a function of range. Such a concept would result in the use of a relatively large firing region for short ranges as compared to long and, consequently, a shorter delay in firing at short ranges. This seems in line with common sense dictates, since at shorter ranges the enemy is a greater threat, and it is of more importance to shoot fast. Note also that it is apparent the firing region should be larger in azimuth than in elevation.

Target Size and Firing Region

Before leaving the question of the dimensions of the firing region, it would be well to consider again the question of target. The use of a 7½ foot square target seems to have become reasonably standard for the purposes of evaluation. However, it may not be the best size target to consider for design purposes. Since tanks frequently attack targets of larger and smaller size than 7½ foot square, it may be desirable to provide a gunner adjustment to the size of the firing region. For instance, a firing region which would give optimum results against a 7½ foot square target would not be best against a smaller target (e.g., a bunker or a partially obscured tank). The effect of this choice would be to sacrifice some time for accuracy (by a smaller firing region) only in the cases where more accuracy is needed because of the smaller target presented.

Frequency and Duration of Delay in Firing

So far, discussion has centered around the effect of the three-switch proposal on accuracy of fire. It has been seen that firing becomes more accurate as the firing region is reduced in size. But what of the delay in firing brought about by the smaller size? Table 8 gives an indication of how frequently firing would be delayed. Some information on the length of the delay is also desired.

To have obtained reliable data on the length of the delay in firing and to have separated the influence of such variables as ground characteristics, tank weight and suspension system, speed, stabilization system, etc., would have constituted a prohibitive amount of labor for the purposes of this report. Some perturbation strip camera records under a variety of conditions have been analyzed, however, to obtain the time delay in firing for these runs. A brief description of the data, the method of analysis, and results will now be given. Special attention should be paid to the limitations of the data.

A. The Data

Strip camera records from tests at Aberdeen Proving Ground were obtained. These records (from boresighted cameras) show the angular displacement of the gun from a reference point in 1/18 of a second intervals. The prepared course over which the tank was run will have considerable influence on the applicability of the data to field conditions. For a description of the course, the reader is referred to the Aberdeen publication.[31]

B. The Method of Analysis

For each chart, different firing regions were assumed. A tally was then made of the specific time intervals in which the gun was within the firing region both for elevation and for azimuth. The number of 1/18th of a second intervals that the gun was within the firing region was tallied a zero delay. For each 1/18th second interval for which the gun was not in the firing region, the time period until the next time the gun entered the firing

[31] APG TT2-645/2, "British Centurion II Tank, Stabilizer Test," SECRET. (Ref. 8).

region was tallied. After all of the run had been covered in this fashion, a frequency distribution of time lapse to fire was obtained. This method of analysis assumes that the gunner is equally likely to see his target at any point (or 1/18th of a second interval) along the length of his run. Simple exponential curves were fitted to the frequency distributions obtained by the above procedure. This curve was for many of the distributions a good fit, but for other distributions the fit was only approximate.

C. Limitations of the Data

The data given on the time delay in firing is, as indicated, applicable only to the given situation under which it was gathered. It applies strictly only to the particular conditions of tank speed, terrain characteristics, stabilization strength and system, and other unique characteristics of the tank under study. This data, therefore, has no general applicability but is meant to give only a rough order of magnitude figure.

It would, of course, have been desirable to analyze a large number of strip camera records in the manner described above. However, this was impossible owing to the lack of records in any large number, particularly for runs under field conditions. Even if they had been available, the computational labor involved in such a venture would have been enormous.

The above restriction is particularly applicable to the data for the Centurion tank because of the type of prepared course over which it was run. In general, the more frequent the shocks given to the tank the smaller the delay in firing.

D. Results

Table 8 gives the average time lapse in firing for a variety of tanks, courses, and speeds. One should notice the quite large variation in this average from one situation to another. Figures 26 to 29 present the fitted exponential curves for some of these situations. It is to be noted that time lapse for firing is generally smaller for the Centurion than for the M24. This situation may be due to the non-comparability of the data, since the data for the M24 was not taken on the same prepared course over which the M4 and the Centurion were run.

As might be expected, the delay in firing is less for a straight course than for a zigzag course. Note also that the time delay is less for the larger firing regions.

Because of the limitations of the data expressed above, the results should be accepted as giving only very rough indications. Should the three-switch proposal be given favorable consideration, it is recommended that further study along the above-indicated lines take place.

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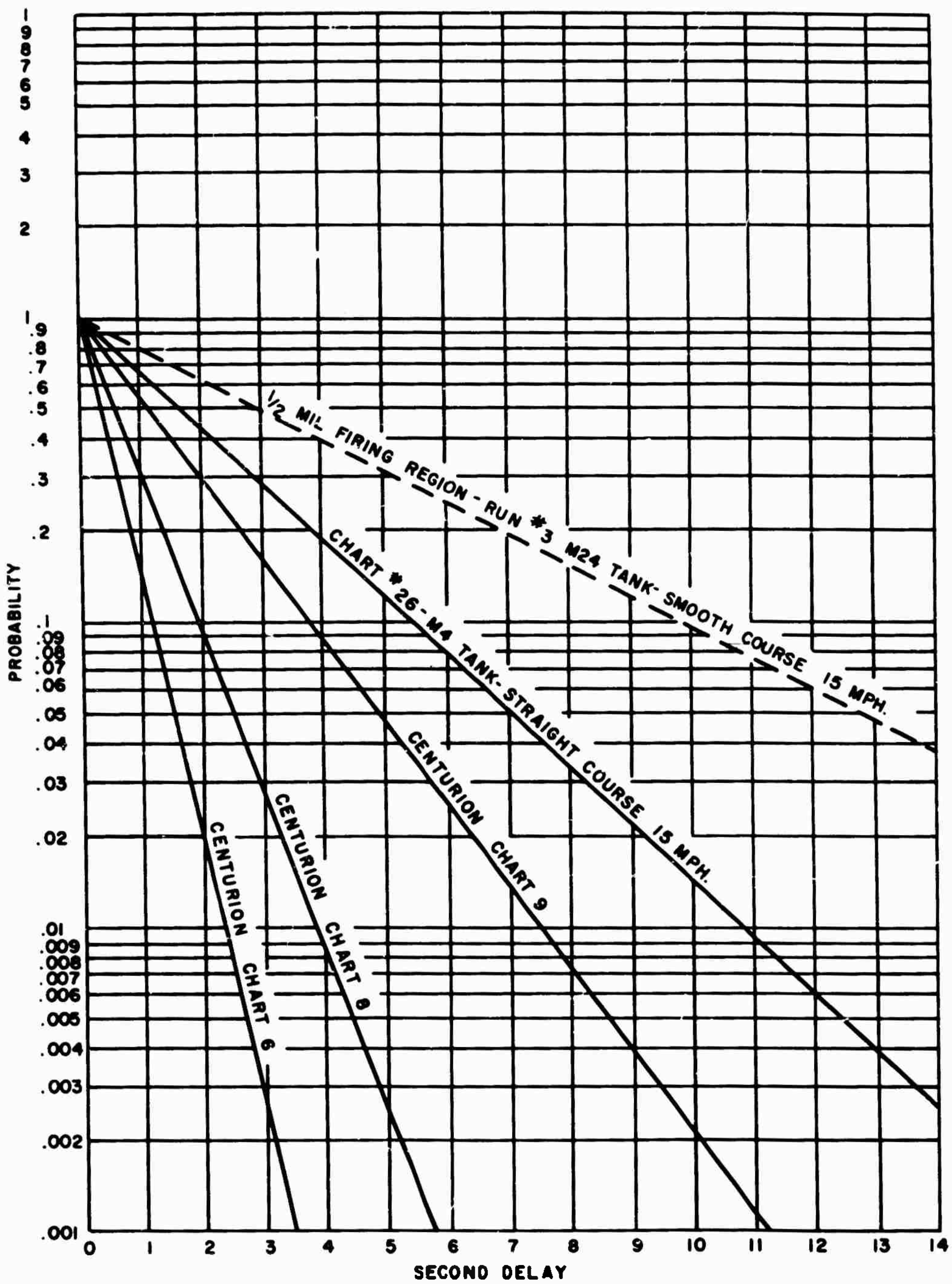

FIGURE 26. *Probability of More Than t Seconds Delay in Firing —with ½ Mil Firing Region. Exponential Curve Fitted to Centurion Data from Aberdeen—Straight Course 15 mph.*

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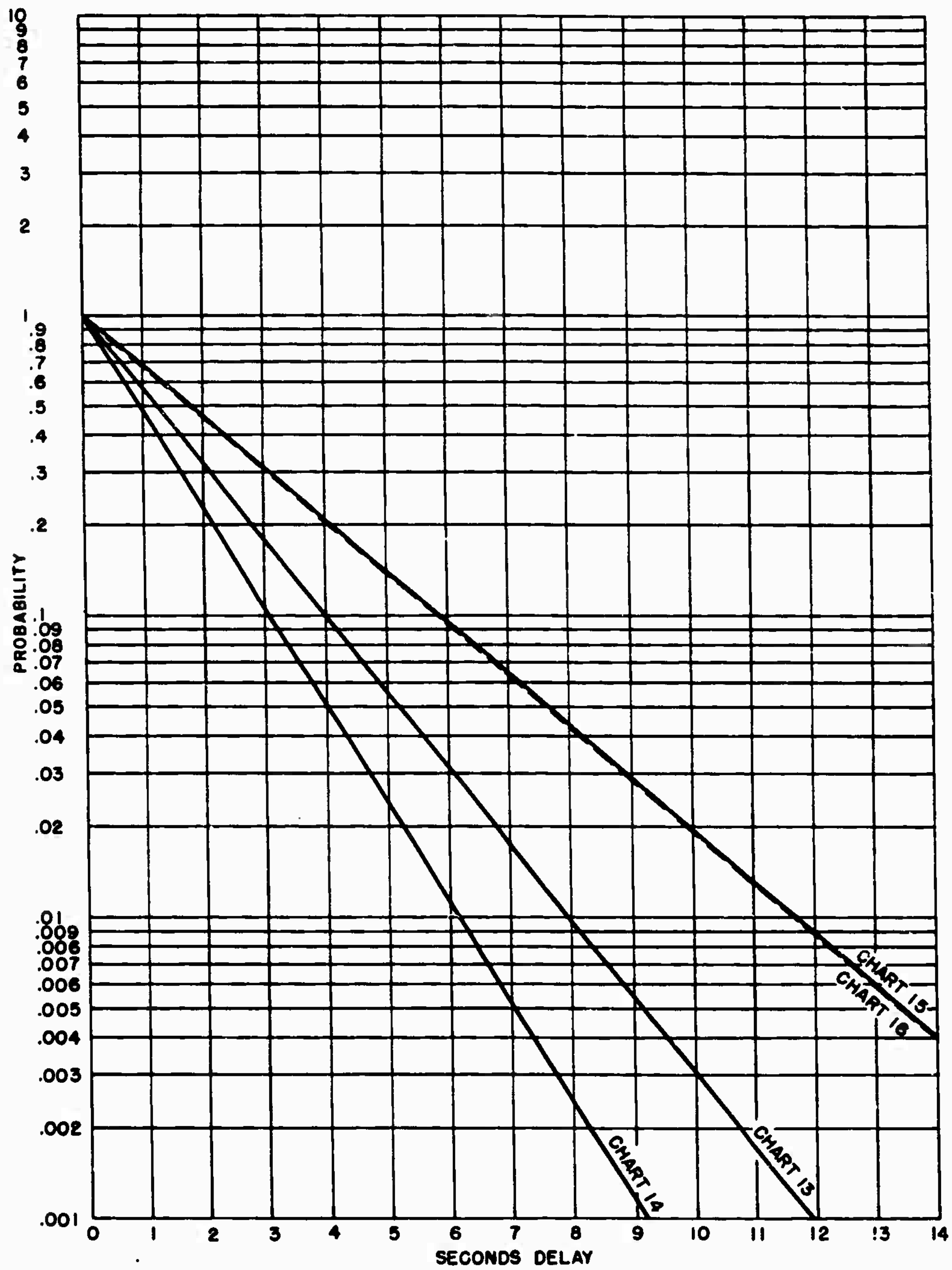

FIGURE 27. *Probability of More Than t Seconds Delay in Firing —with ½ Mil Firing Region. Exponential Curve Fitted to Centurion Data from Aberdeen—Zigzag Course 15 mph.*

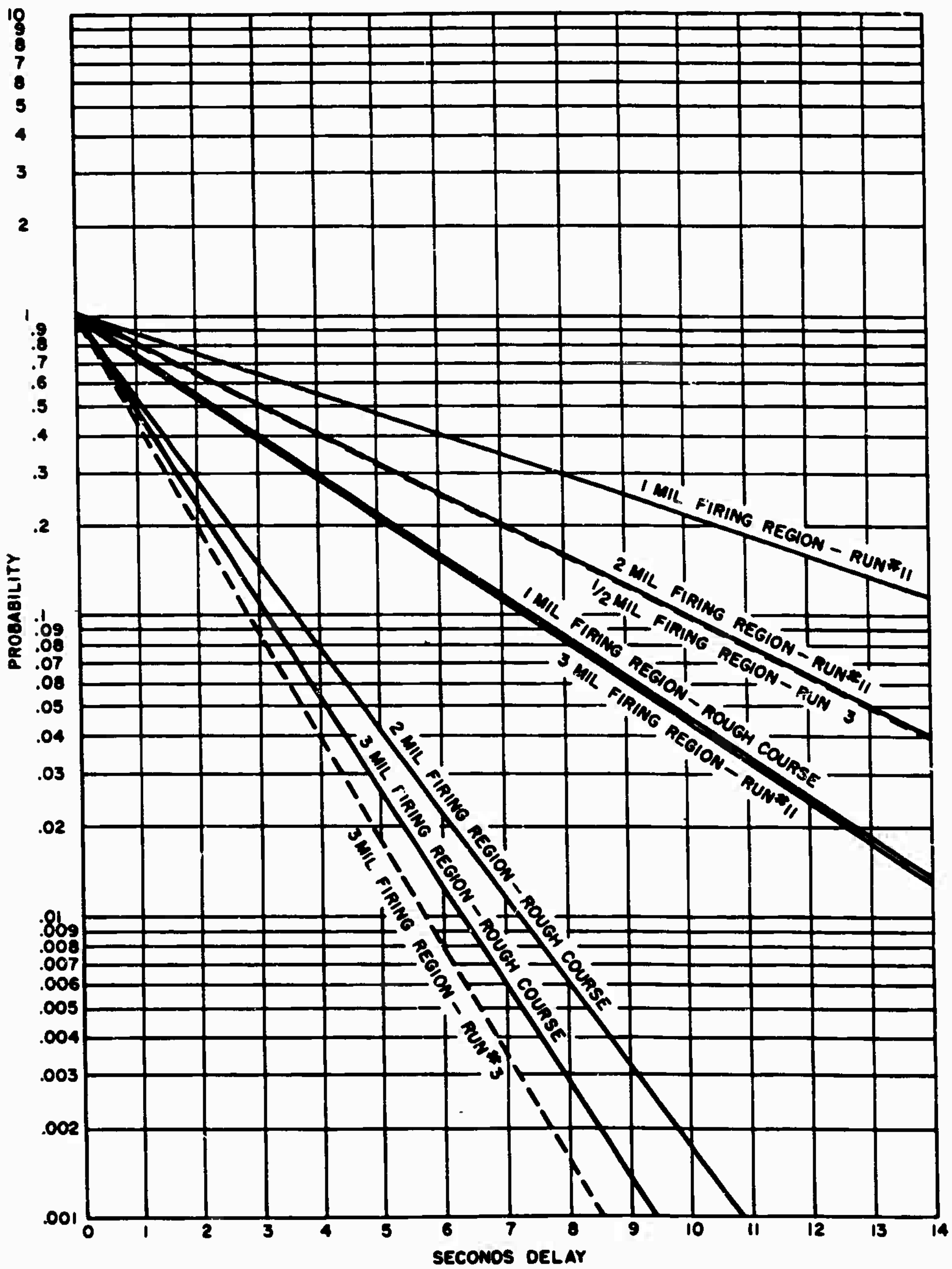

FIGURE 28. *Probability of More Than t Seconds Delay in Firing —15 mph. Exponential Curve Fitted to M24 Data—Smooth Course-Run #3, Smooth Course-Run #11, Rough Course-Run #6.*

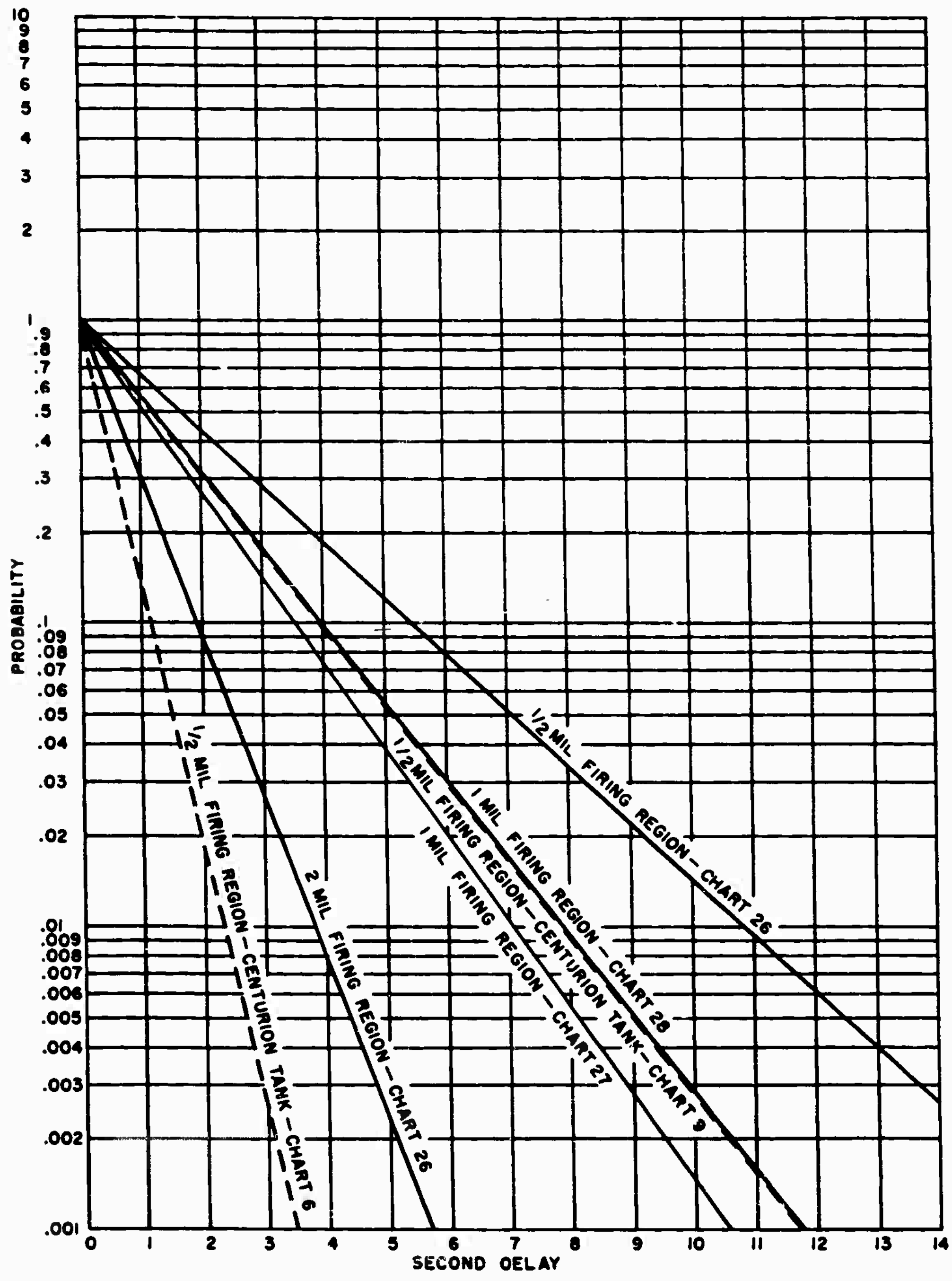

FIGURE 29. *Probability of More Than t Seconds Delay in Firing. Exponential Curve Fitted to Aberdeen Data—M4 Tank—Straight Course 15 mph.*

TABLE 8.

Average Time Delay in Seconds in Firing for Selected Tank Runs at 15 mph under the Three-Switch Proposal.*

			Firing Region (mils)**		
Tank	Chart	Course	½	1	2
Centurion II	6	Straight	.5	...	...
	7	Straight	...	.2	...
	8	Straight	1.1	...	...
	9	Straight	1.7	.7	...
	13	Zigzag	1.8	.9	...
	14	Zigzag	1.7	1.0	...
	15	Zigzag	2.7	1.6	...
	16	Zigzag	2.5	...	...
M24		Smooth Straight	4.3	3.1	...
	6	Rough Straight	...	3.3	1.5
	11	Smooth Straight	...	6.5	4.3
M4	26	Straight	2.3	...	.8
	27	Straight	...	1.5	...
	28	Straight	...	1.7	.8

* For an exponential curve, about 64% of the cases are less than the average and about 83% are less than twice the average.

** Blank spots denote firing regions for which calculations were not done.

V. SIGHT STABILIZATION ONLY

The suggestion to stabilize only the sight holds great promise, if used along with the three-switch proposal. In this section the reasoning which leads to this conclusion will be stated. Because of the large mass of the gun it is expected that great savings in power requirements would accrue if stabilization of the gun were not necessary. The stabilization equipment will become less costly, less bulky, and easier to maintain and supply.[32]

In this section the accuracy of the following systems will be compared:

(a) The present system wherein the gun and the sight are stabilized as one unit.

(b) The gun and sight are separately stabilized.

(c) The gun and sight are separately stabilized and a firing region is used for the gun.

(d) Only the sight is stabilized, and a firing region is used for the gun.

It will be shown that system (b) is superior in accuracy to system (a); that system (c) is superior in accuracy to system (b); and that systems (c) and (d) are of comparable accuracy.

At the outset, a simplifying assumption will be made that the gun fires and the shell leaves the muzzle at the instant the three switches are thrown (when the gun crosses the boundary of the firing region, or when it fires without delay). The consequences of the assumption will be considered later in this section.

Table 9 gives the single-shot probability of hit for a variety of circumstances, subject to the assumptions given in the footnote to the table and in the above paragraph. Probabilities for two values of σ_S are given. The larger σ_S is the estimated value for a system in which it is assumed that no additional power is available to stabilize the sight more tightly than is now the case. The smaller value of σ_S is the minimum which is assumed to be possible with more power available for sight stabilization. The smaller values of σ_S would apply to a system of limited gun stabilization or no gun stabilization.

Also shown on Table 9 are probabilities for several values of σ_G. The two lower values (1 and 3 mils) represent roughly those applicable to moderate terrain and tank speed and to rough terrain and high tank speed, respectively, when the gun is as tightly stabilized as it is now. The larger values represent a less tightly stabilized gun. The largest value is assumed to apply to a non-stabilized gun when the tank is going over moderate terrain at moderate speed.

The third variable shown on Table 9 is the dimension of the firing region, (A). Probabilities are given for four values of (A) from ½ to 2 mils. The value of $A = \infty$ is equivalent to a system in which there is no firing region.

There are several things of importance to the problem, which are revealed by a study of this table:

(1) An increase in accuracy (from .92 to .98 for 400 yds; from .46 to .60 for 700 yds; and from .24 to .28 for 1000 yds) is obtained by the combination of tight sight

[32] It should be borne in mind throughout this section that, where the gun is spoken of as not stabilized, the intended meaning is that the gun is stabilized only to the extent required for its average position to follow the sight gyro. Instantaneous gun perturbations can be of the order obtained when no stabilization equipment is used so long as gun drift is eliminated.

TABLE 9.

Probability of Hitting a 7½ Ft. Square Target from a Moving Tank when Gun is Stabilized to Various Degrees of Tightness (σ_G) and Sight is Stabilized to Various Degrees of Tightness (σ_S), and where the Gun is Equipped with a Firing Region ($A < \infty$) and where it is Not ($A = \infty$).*

	σ_S = ¾ mil					σ_S = ¼ mil				
A (mils) / σ_G (mils)	½	1	1½	2	∞	½	1	1½	2	∞
400 Yds Range										
9	.98	.95	.91	.86	.07	.995	.98	.96	.92	.07
7	.98	.95	.91	.86	.12	.995	.98	.96	.92	.12
5	.98	.96	.91	.87	.21	.995	.98	.97	.93	.22
3	.98	.96	.92	.88	.46	.995	.99	.97	.93	.49
1	.98	.96	.94	.93	.93	.995	.99	.98	.97	.96
700 Yds Range										
9	.58	.53	.43	.30	.02	.63	.60	.50	.32	.02
7	.58	.53	.43	.31	.04	.63	.60	.50	.33	.04
5	.58	.53	.44	.33	.07	.63	.60	.50	.35	.07
3	.58	.53	.45	.36	.16	.63	.60	.51	.39	.17
1	.58	.58	.51	.47	.46	.63	.61	.56	.54	.52
1000 Yds Range										
9	.28	.23	.16	.12	.01	.33	.28	.18	.13	.01
7	.28	.23	.16	.13	.02	.33	.28	.19	.14	.02
5	.28	.23	.17	.14	.03	.33	.28	.19	.15	.03
3	.28	.24	.18	.15	.07	.33	.28	.21	.17	.08
1	.28	.25	.23	.22	.21	.33	.30	.27	.25	.24

* In the calculations for the above table, the following assumptions have been made:
(1) Errors in azimuth are equal to those in elevation for sight error, gun error, and stationary firing error.
(2) The stationary firing error is 1/5 mil.
(3) The mean range estimation error is 17%.
(4) The gun is a 75mm firing AP shot (muzzle velocity 2030 f/s).
(5) The firing region is a square.
The values are calculated from the following formula as given on page 37: $P^* = (1 - P_D)\,P_I + P_D P_F$.

stabilization, no gun stabilization, and a firing region, in preference to the present system of sight and gun stabilized together without a firing region.[33, 35]

(2) For firing regions of A = ½ mil and A = 1 mil, the non-stabilized gun with firing region and tightly stabilized sight is more accurate than the stabilized gun with firing region and more loosely stabilized sight. In the case of A = 1½ mil, these two

[33] The first of these is represented by σ_S = ¼ mil; σ_G = 9 mil, and A = 1 mil, while the second is represented by σ_S = ¾ mil, σ_G = 1 mil, and $A = \infty$.

systems are of approximately the same accuracy. For $A = 2$ mils their relative accuracy is reversed.[34, 35]

(3) When a firing region is used, the probability of hit is relatively insensitive to the magnitude of gun perturbations, except for values of σ_G, which are small in comparison to (A).

(4) The accuracy of any system with a firing region is rather sensitive to the size of the firing region. In general, the probability of hit with a firing region approaches that without a firing region as (A) approaches $2\,\sigma_G$.[36]

Study of Table 9 suggests the adoption of the following system: tight sight stabilization and no gun stabilization with firing region of $A = 1$ mil. While a value of $A = \frac{1}{2}$ mil would further increase the accuracy, the consideration favoring the choice of $A = 1$ mil was the lessened time delay in firing (See Table 8). It is possible, however, that further study of angular velocities of an unstabilized gun will indicate a value of $A = 1\frac{1}{2}$ mil to be more favorable. Furthermore, additional study of angular velocities of an unstabilized gun may indicate the desirability of limited gun stabilization rather than no gun stabilization.

The Effect of the Simplifying Assumption

The effect of the simplifying assumption made earlier in this section will now be considered.

The calculations cited in the early part of this section assume that the shell leaves the muzzle of the gun at the instant the gun crosses the boundary of the firing region.

TABLE 10.*

Time Lag in Firing

Element	Time-Seconds: Average	Time-Seconds: 90% Range
Simple reaction time (of gunner)	.285	.24 to .34
Reaction time while tracking (of gunner)	.407	.30 to .48
Lag in firing mechanism**	.104	.095 to .130
	.511	.44 to .60

* From Ref. 2, "Capacities and Limitations of Moving Fire with Gyrostabilizer" page 9.

** Foot Button = .042, solenoid and mechanical linkage = .062.
In the firing region proposal, only the lag of the firing mechanism is present. The foot button time is *not* included. To this figure should be added .005 to .015 second for the shell to travel the length of the gun. It is seen, therefore, that in order to keep the time lag to a figure approximating .015 second, electric, rather than mechanical, switching and priming is necessary.

[34] Compare the probability for $\sigma_G = 1$ mil and $\sigma_S = \frac{3}{4}$ mil with that for $\sigma_G = 9$ mil and $\sigma_S = \frac{1}{4}$ mil, using the same value for (A).

[35] In the calculations of this report, the firing region was taken to be a square for computational convenience, but it is shown elsewhere in this report that such an arrangement is not optimum. Thus, the probability of hit under the three-switch proposal can be increased beyond what is shown in this report. Further study of this factor is indicated.

[36] Theoretically, the probability with a firing region approaches the probability for $A = \infty$ as an asymptote.

However, there is a time lag of the order of .015 second (see Table 10), and this lag is large enough to have influence on the probability of hit.

In order that confusion might not arise because of similarity of names, this time will be referred to as a time *lag*, whereas the one resulting from the gun being outside the firing region when the gunner's switch is thrown will be referred to as a time *delay*.

The influence of this time lag is to change the angle of elevation of the gun, raising (or lowering) the trajectory an amount for elevation equal to $\triangle h$ where:

$$\triangle h \approx \pm \omega\tau \text{ (in mils)} \tag{9}$$

ω = angular velocity of the gun in radians/sec

τ = time lag in seconds.

For the effect on azimuth error, the same formula also holds.[37]

The effect of the simplifying assumption (that $\tau = 0$) in the event of delayed firing will now be considered. It will be assumed that (1) the gun is practically certain to move toward the center of the firing region in the event of delayed firing,[38] (2) the distribution of angular velocities is exponential with a mean of $\overline{\omega}$,[38] and (3) it is desired that the time lag result in increased accuracy for 85% of the delayed firings (over the accuracy attainable when $\tau = 0$).[39]

Since ω is assumed to be exponentially distributed, 85% of the angular velocities are expected to have values less than 2 $\overline{\omega}$; hence 85% of the angular distance traveled by the gun during the time period τ is less than 2 $\omega\tau$.[40] In order that improvements in accuracy (over the case of $\tau = 0$) will result 85% of the time, it is therefore necessary that

$$2A > 2\omega\tau \tag{10}$$

$$A > \omega\tau$$

The foregoing tabulation of M24 runs indicates that a value of $\overline{\omega} = 60$ is likely the largest value to be encountered in azimuth for a non-stabilized gun. Since the value of τ is considered to be in the neighborhood of .015 second, any value of $A > .9$ mil will likely result in greater accuracy for the three-switch proposal than is indicated by the analysis

[37] See derivation at end of this section.

[38] Tabulations of gun movements in elevation and azimuth for the M24 tank (whose gun is stabilized only in elevation) indicate that, within a 1/10 second interval, the gun is 8 times as likely to continue in the same direction as to reverse direction. With consideration given to the fact that the actual time lag is of the order of magnitude of .015 second rather than 0.1 second, it would appear reasonable to assume that the gun would be even less likely to change direction than indicated. However, the data available on gun perturbations is not sufficiently detailed to enable tabulations for smaller intervals than 0.1 second. The same charts indicate that the distribution of angular velocities is approximately exponential with the following averages:

		Average Angular Velocity (mils/sec)
Rough course	6 mph	18
Rough course	14 mph	61
Zigzag	2 mph	20
Zigzag	6 mph	39

[39] It is felt that more work is necessary to define optimum criteria here. The above-stated criterion is arbitrary. It implies that 15% of the firings will be less accurate.

[40] This statement carries an implied assumption that τ is not a random variable.

which assumes $\tau = 0$ for the case of no delay in firing.[41]

The effect of the simplifying assumption (that $\tau = 0$) in the event of no delay in firing (i.e., if the gun is inside the firing region when fired) will now be considered.

Since the number of random variables already considered (stationary, range, gun, and sight error) are large enough to assume that their sum is normally distributed, a normal distribution of the sum which includes the gun travel during the interval of $\tau = .015$ second may also be assumed. The standard deviation of this distribution is (for elevation) $(\sigma T_E'')^2 = \sigma_{A_E}{}^2 + \sigma_{R_E}{}^2 + \sigma_{S_E}{}^2 + K\sigma_G{}^2 + \sigma_h{}^2$
where K is the factor due to truncating, h is the distance the gun moves during the time τ, and the other factors are as previously defined. The expression for azimuth is similar, with $\sigma_R{}^2$ excluded. The accuracy when $\tau > 0$ is, therefore, less than the accuracy when $\tau = 0$.[42] There are at least two factors, however, which lessen the importance of this fact. These are: (1) the percentage of the shots which would fire without delay are in the great minority. The weight of this decreased accuracy is therefore much smaller than the weight of the increased accuracy for the case of delayed fire[43] so that the contribution to over-all accuracy is less. (2) the improvement in accuracy for the case of delayed fire is at least of comparable magnitude to the lessening of accuracy in the case of no delay in firing.

Based upon information currently available, therefore, it is anticipated that the simplifying assumption would not invalidate the recommendations. It is realized, however, that the data on angular velocity of a non-stabilized gun is rather sketchy. In particular, no data was found for perturbations of a non-stabilized gun for elevation, and only a limited amount was available for azimuth. Further study on this point is therefore recommended.

Should further study indicate that the angular velocities to be expected are much higher than is indicated in this report, limited stabilization of the gun would be indicated. In any event, however, other recommendations would still hold, since the amount of power necessary for limited stabilization would be only that required to reduce the aver-

[41] Since perturbations induced in elevation may be larger than those in azimuth, the required value of A may be larger for elevation. Or, for the same value of A, the probability of improvement in accuracy will be less. Since no data on the movements in elevation for a non-stabilized gun was available, this point has not been investigated.

[42] This statement carries an implied assumption that τ is not a random variable. Since $h = \omega\tau$ then $\sigma_h{}^2 = \tau^2\sigma_\omega{}^2$ and since $\bar{\omega} = \sigma_\omega$ then $\sigma_h{}^2 = (\tau\bar{\omega})^2$. Because ω may take on both positive and negative values with respect to the center of the firing region (while in the case of delayed fire ω was assumed directed toward the center), the distribution of ω is assumed to be a different form than exponential. The applicable distribution is assumed to be exponential on either side of $\omega = 0$. By the rules of combining frequency distributions, the variance of the combined distribution is the weighted average of the variance of the individual distributions (where the weights are the ratio of frequency of the distribution to the total frequency of both distributions). Since the frequency on each side of $\omega = 0$ is the same, the variance of the combined distribution is $(\tau\bar{\omega})^2$.

[43] The percentage of no delay in firing ($1-P_D$) as a function of σ_G and A is as follows:

A	½	1	1½	2
σ_G				
9	.0442	.0884	.1323	.1757
7	.0568	.1130	.1695	.2251
5	.0797	.1585	.2358	.3108
3	.1326	.2610	.3829	.4950
1	.3829	.6827	.8664	.9545

age angular velocities to the order of magnitude of $\bar{\omega} < 60$ mils/sec. in both azimuth and elevation, and such power requirements are still expected to be small compared with that now being used.

Depending on the results of further study, it may also be desirable to limit the ability of the gun to fire by using a fourth switch, which would permit firing only when instantaneous velocities were below a given figure.

Analysis: Effect of Gun Travel During Period of Exploding Primer to the Instant the Shell Leaves the Muzzle of the Gun

1. Effect on Elevation Error

$$\Delta h = r\left[\tan(\theta + \omega\tau) - \tan\theta\right] = r\left[\frac{(\tan\omega\tau)\,(4\bar{V}^4 + g^2r^2)}{4\bar{V}^4 - 2\bar{V}^2\, gr\, \tan\omega\tau}\right]$$

after trigonometric manipulation and substitution of $\tan\theta = gr/2\bar{V}^2$.

Since $\omega\tau$ is a small angle (in radians), so that $\dfrac{\tan(\omega\tau)}{\omega\tau} \approx 1$,

$$\Delta h = r\left[\frac{\omega\tau 4\bar{V}^4 + \omega\tau g^2r^2}{4\bar{V}^4 - 2\bar{V}^2\, gr\omega\tau}\right].$$

$$O = r\tan\theta - \tfrac{1}{2}gt^2 = r\tan\theta - \tfrac{1}{2}g\left(\frac{r}{\bar{V}}\right)^2$$

$$\tan\theta = (\tfrac{1}{2}gr)/\bar{V}^2$$

Similarly for a gun traveling downward

$$\Delta h = -r\left[\tan(\theta - \omega\tau) - \tan\theta\right] = -r\left[\frac{\omega\tau(4\bar{V}^2 + g^2r^2)}{4\bar{V}^4 - 2\bar{V}^2 gr\omega\tau}\right]$$

since $\dfrac{\tan\omega\tau}{\omega\tau} \approx 1$ and $\tan\theta = (\tfrac{1}{2}gr)/\bar{V}^2$.

$\Delta h \approx -r\,(\omega\tau)$ since both g^2r^2 and $2\bar{V}^2\, gr\omega\tau$ are small compared to $4\bar{V}^4$.

2. Effect on Azimuth Error

The formulas for azimuth errors are obtained directly from the triangular relationship:

$\Delta h = r\tan\omega\tau \approx r\omega\tau$	for positive angles, and
$\Delta h \approx -r\omega\tau$	for negative angles.

In the above notation:

Δh = distance of the shot from a point target
r = range
θ = angle of elevation
ω = angular velocity of gun during the period, in radians/sec.
τ = time lag, in seconds
$\bar{V}$ = average velocity of shell in ft./sec.
g = gravitational constant (32.2 ft./sec.^{2}).
t = time of flight of shell

VI. RATE OF FIRING

Before discussing tank battles, it is considered desirable to give some data on the time required for a tank to shoot. Of necessity, experimental data of this type is confined to the present system of sight tied to gun.[44]

Firing from a Halt

The time required to halt, engage the target, and fire the first shot appears to be of the order of 14-18 seconds (for a tank moving at 15 mph).[45] The time required to fire the second shot after the target has been engaged is about 7 seconds.

Moving Fire with Present Stabilization System

Test data indicates that the average time to fire a shot from an M4 tank moving at 10 mph is from 8-10 seconds at 500 yds and from 10-15 seconds at 1000 yds. This tank was not stabilized in azimuth, and as APG points out in the report on the T41,[46] "Devotion of so much time by the gunner to azimuth correction influences the rate of fire and accuracy of fire in the elevation plane."

Table 11 summarizes the rate of fire data given by APG in this same report.

TABLE 11.

Rates of Fire by the Centurion, T41 and M24 Tanks

Rounds/Minute*

Condition	Tank		
	Centurion	T41	M24
Over-all Average	4.62	4.44	3.14
Straight Smooth Course			
6 mph	4.66	5.91	3.14
15 mph	6.70	5.00	3.33
Straight Rough Course			
6 mph	5.30	4.70	3.00
15 mph	3.95	4.42	
Zigzag			
6 mph	4.10	3.20	
15 mph	3.30	2.93	
Circular			
6 mph	4.51	2.56	

* In the calculations for rounds per minute the first round fired on each run was disregarded to eliminate variations in starting procedure.

[44] Subsequent to the completion of this report, Project Stalk was conducted by APG. The purpose of this project was to measure the times discussed in this section, among others, for a variety of tank and fire control combinations. The reader is referred to "An Assembly of Project Stalk Data" by F. I. Hill, et al BRL Memorandum Report No. 745 (Confidential) January 1954.

[45] "Tank, 76mm gun, T41 Integrated Fire Control System and Comparative Tests of Tank Gun Stabilizer," SECRET. (Ref. 9).

[46] Fort Knox, Ref. 2.

While it is not known to what ranges the data given in Table 11 applies, the tests of these tanks were generally from 800 to 1200 yards, and the firing times are comparable to those given in the Fort Knox report, Ref. 2, i.e., of the order of 9-19 seconds for a straight smooth course, depending on the tank. With only the Centurion and the T41 tanks considered, the following times seem representative:

9-13 seconds for smooth straight course,
11-15 seconds for rough straight course,
15-20 seconds for zigzag course.

In general, the time required to fire from the Centurion is somewhat less than from the T41. This is expected to be a result of tighter stabilization of the gun of the Centurion and of the Centurion's greater weight, which means a more stable loading platform.

It should be noted that, while the stabilized tank may gain some additional protection by evasive maneuvers, it also loses in terms of firing. Since a halted tank (after the first shot) requires only 7 seconds to fire, the halted tank will get two shots to the stabilized tank's one shot, and should the stabilized tank attempt to maneuver, it will be forced to increase this disadvantage to 3:1 (3 shots for the halted tank to one of the stabilized).

Moving Fire with Proposed Alternative Systems

Since no data is available on any of the proposed alternatives with respect to firing rate, it is possible to give only general statements as to their ranking with the present system.

The system which calls for separate stabilization of the gun and sight (without firing region, or automatic loading) is expected to fire at the same rate as presently. If a firing region is used (along with as tight gun stabilization as at present), the time required to shoot will be increased by the amount indicated in Figures 26 to 29. A tank so equipped will then (with high probability) fire the first shot but add, on the average, 2 to 4 seconds to the time required for subsequent shots. If automatic loading is used in addition to a firing region, the time to fire shots after the first is expected to be comparable to that for the halted tank (in the neighborhood of 7 seconds).

The system which calls for stabilization of the sight only will likely fire all shots subsequent to the first at a slower rate than presently, unless equipped with an automatic loader. This is expected because of the rapid movements of the gun, which would make manual loading very difficult, if not impossible. On the other hand, if automatic loading is used, the time is expected to be slightly less than the system requiring separate stabilization of the sight and gun with firing region and automatic loader. This is expected because of a shorter time delay owing to the higher angular velocities of the gun.

Summary

In summary, the following is a ranking of the proposed systems in order of expected length of time to fire all shots (subsequent to the first for a moving tank). The shortest time is given first:

Rank		With Automatic Loader	With Firing Region	Estimated Time (Seconds)
1	Separate stabilization of gun and sight	x		
1	HALTED TANK			7
1	Sight stabilization only	x	x	
2	Separate stabilization of gun and sight	x	x	
3	Separate stabilization of gun and sight			
3	GUN AND SIGHT STABILIZED TOGETHER			13
4	Separate stabilization of gun and sight		x	
5	Sight stabilization only		x	

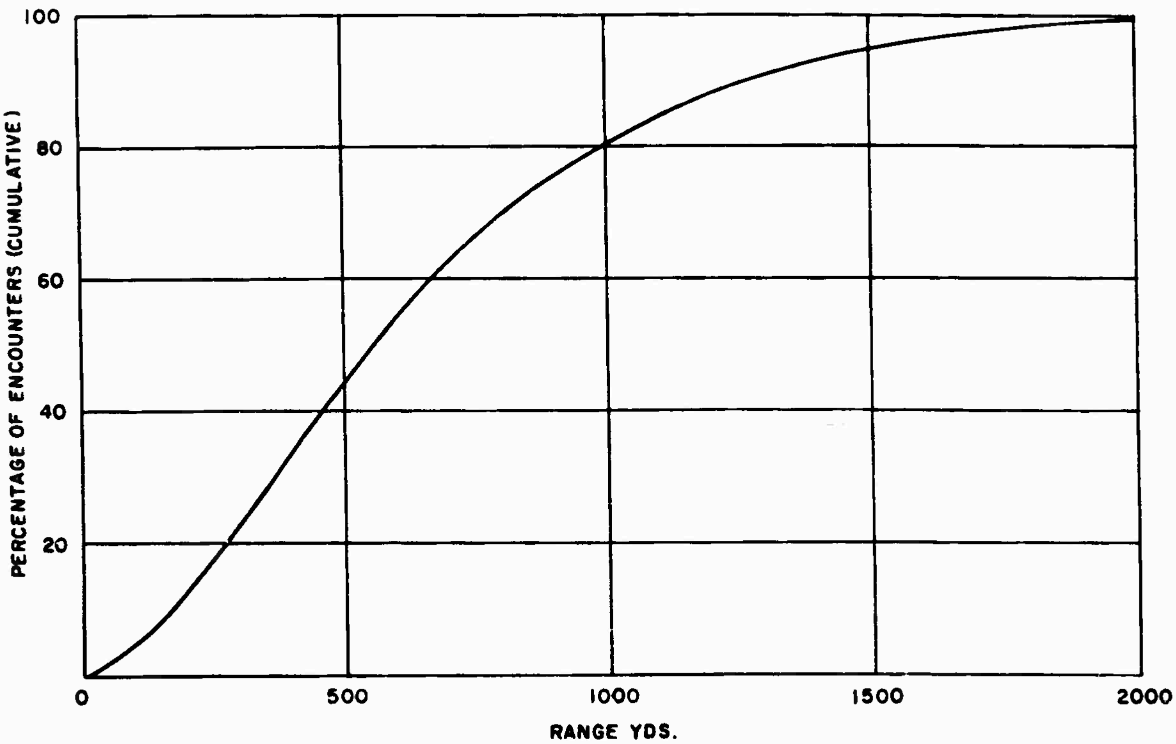

FIGURE 30. *Frequency of Encounters between Tanks and Anti-Tank Weapons (including Tanks) Firing AP Ammunition in N. W. Europe during World War II.**

* Based on a chart from Aberdeen Proving Ground.

VII. RANGE FINDING WITH MOVING FIRE

This report is not intended as an evaluation of the range finder. However, since the error due to range estimation is the largest single error even for fairly moderate ranges (1000-1500 yds.), it is necessary to discuss the various methods of finding range as they relate to moving fire. The three methods to be considered are: (1) "burst on target" method, (2) optical range finder, and (3) visual range estimation.

The "burst on target" technique, which is used in firing from a halt or in stationary fire, would not be advantageous in moving fire.

The reason "burst on target" works so well with stationary fire is that the major part of the correction (distance from target to burst) is for range estimation. The amount of needed correction for range is relatively constant throughout the engagement and is large compared with the variation of shots due to other errors. Such would not be the case in stabilized fire, however, because a large part of the indicated correction (shell burst to target) would be due to the gun movement, which is variable for different shots of the same engagement. If one were to use "burst on target" technique during moving fire, one would, therefore, be adding to the total error by increasing the variance of sighting error an amount equal to the variance of gun movements. In any event, "burst on target" technique cannot be used unless the sight is very tightly stabilized. The above argument indicates, however, that even then it would not be profitable unless the gun also were tightly stabilized. This, of course, cannot be done with available power.

Since "burst on target" is not expected to decrease the range error, one inquires next into the possibilities of an optical range finder.

The big difficulty involved in using an optical range finder during moving fire is not a matter of accuracy. Indeed, it would appear that the error due to range estimation is reduced by a factor of 2 or 3 when a range finder is used instead of visual range estimation. The objection to using a range finder in moving fire is the amount of time required for its use. Experiments at Aberdeen Proving Ground[47] indicate that about 5 to 15 seconds is required to obtain one range reading with a range finder. Since a principal reason for stabilization is to obtain a time advantage of about 5 seconds at the beginning of the engagement, it appears that the use of a range finder during moving fire defeats one of the main purposes of stabilization.

The same question is now considered in a slightly different light. In order to gain the advantage of the first shot, one accepts stabilization, which increases the variation of shots by approximately 1.5 to 4 mils (depending on certain conditions mentioned elsewhere in this report). One cannot, with consistency, also use a range finder, which places the first shot on, at best, an equal-chance basis and may even guarantee the first shot to the enemy. In addition, throughout most of the range in which tank battles take place less accuracy is gained by the range finder than is given up by stabilization.

A curve presenting the frequencies of encounters between tanks and anti-tank weapons in N. W. Europe during World War II is presented in Figure 30.[48] The reduction in range estimation error due to using a range finder in moving fire is shown in Table 12.

[47] T41 Report, op. cit., Ref. 9.

[48] Data is from Aberdeen Proving Ground.

It is possible that firing from a moving tank might so decrease the enemy tank's accuracy that moving fire gives an advantage without getting the first shot. In this event, the time required to use a range finder may not be a detriment—particularly if the tank is equipped with an automatic loader.

Another possible situation in which the range finder might be more desirable in moving fire could arise if the tactics were to fire the first shot with visual range estimation and to use the range finder for the second shot. Such variations from present practice have not been investigated, although it may be profitable to do so.

With present tactics and equipment, however, it would not appear feasible to use the range finder in moving fire during a tank duel at moderate ranges.

One is, therefore, left with visual means as the most feasible method of estimating range during moving fire. This point should be constantly borne in mind throughout the following section on tank duels.

TABLE 12.

Range Error of Visual Estimation and with a Coincidence Range Finder in Moving Fire.*

Range (Yds)	Standard Deviation (Mils) Visual Estimation (MREE = 20%)	Range Finder (MREE = 7%)	Reduction in Standard Deviation
200	.5	.2	.3
400	.9	.3	.6
600	1.4	.5	.9
800	2.0	.7	1.3
1000	2.5	.9	1.6
1200	3.1	1.1	2.0
1400	3.7	1.3	2.4
1600	4.3	1.5	2.8
1800	5.1	1.8	3.3
2000	5.8	2.1	3.7

* Assuming a 75mm gun firing AP shot (muzzle velocity 2340 ft/sec). The reduction would be less for higher muzzle velocity guns.

VIII. TANK DUELS

In previous sections, certain systems of stabilization were examined from the viewpoint of accuracy. In addition, certain comments were made about the advantages or disadvantages of each system in connection with the time required to fire the first shot and the rate of fire thereafter. It was further shown that, in order to gain an advantage of time, it is necessary to give up some accuracy, and conversely. It is the purpose of this section to consider a method of evaluating the relative importance of time and accuracy as they affect survival chances (or kill probabilities) in the case of a battle between two tanks.

In this examination it is necessary to introduce some simplifying assumptions which will invalidate any comparison with actual field results but which will nevertheless leave an instructive example. It is assumed that:

1. Both tanks see each other at the same time.
2. Both tanks are equally matched in terms of crew proficiency, armor, and armament.
3. Both tanks have a plentiful supply of ammunition.
4. A hit on either tank within a 7½ ft. square area is a kill, and a hit outside this area does not affect the tank or crew.
5. The shot of only one tank is in the air at one time.

The element of time will be considered on a relative basis—that is, the time element will be expressed in terms of (1) which tank gets the first shot, and (2) the number of shots the friendly tank gets per shot of the enemy tank.[19]

The formulas applicable to a variety of combinations of these conditions are shown in Table 13. Their derivation is presented in this section below. By following a similar derivation, formulas for a variety of other circumstances may be obtained.

TABLE 13.

Summary of Tank Battle Formulas—assuming that the friendly tank gets the first shot and that each shot of each tank is fired with the same accuracy as its first shot.

(Section VIII)

Refer to Case	Ratio of shots by friendly tank to those of enemy after the first shot	Survival Probability*
3	1/3	$\frac{F}{1-(1-F)(1-E)^3}$
2	1/2	$\frac{F}{1-(1-F)(1-E)^2}$
1	1	$\frac{F}{1-(1-F)(1-E)}$

* The accuracy of the enemy tank (E) may change from system to system because of differing ability of the friendly tank to take evasive action without affecting its own accuracy. For example, E is expected to be higher for the last system than for the others.
F = probability of hit for friendly tank and E = probability of hit for enemy tank.

[19] The tank duel in which the three-switch proposal is incorporated on the friendly tank has not been included in this report because of the complications which result from the fact that the time delay in firing is in terms of a probability curve.

Incidentally, it should be noted that in the initial series of each derivation the coefficient of F in the ith term of the series gives the probability that the battle will have ended before the ith shot by the friendly tank. Such figures are expected to be useful in the design of an automatic loader—specifically, for determining the required capacity of the loader for anti-tank shells.

A comparison of the formulas given in Table 13 will indicate the relative importance of the three factors on the survival probability of the friendly tank: (a) accuracy of the friendly tank, (b) accuracy of the enemy tank (including the effect of the friendly tank's evasive maneuvers), and (c) relative rates of fire.

In order to aid this comparison, the numerical values for these formulas are presented in Table 14. For example, the table shows that, if both tanks have a single-shot hit probability of .3 and the firing rates are equal, the friendly tank's survival probability is .588. If the friendly tank reduces his rate of fire due to movement so that he gets one shot to the enemy's three, his survival probability is reduced to .395. On the other hand, if his movement also reduces the enemy's single-shot hit probability to .1 while keeping his own the same, then his survival is increased to .613, etc.

Thus, if a tank battle model of the type considered is assumed, the relative effects of accuracy and firing rate are given.

As further illustration of how tank battle models may be used, a comparison is made of the survival probability formulas just given to the survival probability when each tank has an equal chance of getting the first shot and they have equal rates of fire. The survival probability formula for the friendly tank in this case is:

$$P_F = \frac{\frac{1}{2}F\,(2 - E)}{1 - (1 - F)\,(1 - E)}$$

which is assumed to describe the situation when both tanks halt to fire.

It was further assumed that the single-shot probability of both tanks was equal in the "halt to fire" situation and the question was posed: "What must the relative accuracies of each tank be in order that the survival probabilities of the friendly tank in the 'halt to fire' situation equal the survival probabilities when the friendly tank gets the first shot and the relative firing rates are specified?" The answer to this question is given in Figure 31: "Minimum Single-Shot Probability of Hit Which a Friendly Tank Must Have in order to Have Greater than .50 Survival Probability."

Graphs of this type will shed light on the following types of questions:

(1) How much single-shot probability of hit should one be willing to give up in order to increase the firing rate?

(2) How much single-shot probability of hit should one be willing to give up in order to get the first shot with certainty?

(3) How much decrease in the enemy's probability of hit (by the friendly tank taking evasive action) must be obtained if the friendly tank gives the enemy tank an extra shot in order to be able to maneuver? (This information cannot be obtained directly from this graph, but can be from another based on the same equations.)

Extensions of Tank Duels

This section on tank duels is meant to be indicative only of the kind of analysis which can be carried out by means of this technique.

The tank duels so far discussed have been very simplified, and their use is suggested for only very limited purposes. The same sort of analysis may be extended to obtain order of magnitude answers to the following problems:

TABLE 14

Survival Probability in a Tank Battle for Various Values of Accuracy (Single-Shot Hit Probability) of Friendly Tank (F), Enemy Tank (E), and Ratio of Rate of Fire of Friendly Tank to Enemy Tank

F	.10			.20			.30			.40			.50			.60			.70			.80			.90		
E	1/1	1/2	1/3	1/1	1/2	1/3	1/1	1/2	1/3	1/1	1/2	1/3	1/1	1/2	1/3	1/1	1/2	1/3	1/1	1/2	1/3	1/1	1/2	1/3	1/1	1/2	1/3
.1	.526	.369	.291	.714	.568	.478	.811	.693	.613	.870	.779	.711	.909	.840	.785	.938	.888	.849	.959	.925	.895	.975	.954	.937	.99	.98	.97
.2	.357	.236	.186	.555	.409	.338	.683	.543	.468	.770	.649	.578	.834	.735	.673	.883	.807	.754	.921	.866	.827	.952	.918	.892	.980	.962	.95
.3	.270	.180	.145	.454	.329	.275	.588	.457	.395	.690	.566	.504	.769	.663	.603	.834	.748	.696	.886	.820	.780	.930	.888	.859	.968	.946	.932
.4	.218	.148	.124	.384	.281	.242	.528	.402	.354	.625	.510	.458	.714	.609	.560	.790	.701	.657	.854	.785	.749	.909	.862	.836	.958	.934	.92
.5	.182	.129	.113	.333	.250	.222	.461	.364	.329	.572	.471	.433	.666	.571	.533	.750	.667	.633	.824	.757	.727	.889	.842	.821	.948	.923	.912
.6	.156	.117	.106	.294	.229	.211	.417	.338	.314	.526	.443	.416	.625	.544	.517	.715	.642	.617	.795	.736	.713	.870	.826	.810	.937	.915	.906
.7	.137	.109	.102	.263	.216	.204	.380	.320	.306	.488	.423	.408	.588	.523	.507	.682	.623	.607	.770	.720	.707	.851	.815	.805	.927	.908	.903
.8	.122	.104	.101	.238	.206	.202	.349	.308	.302	.455	.410	.402	.556	.510	.502	.653	.611	.603	.744	.709	.702	.834	.808	.802	.919	.903	.90
.9	.110	.101	.10	.217	.202	.200	.323	.302	.300	.426	.403	.400	.527	.503	.500	.625	.602	.609	.722	.702	.700	.816	.802	.800	.91	.90	.90

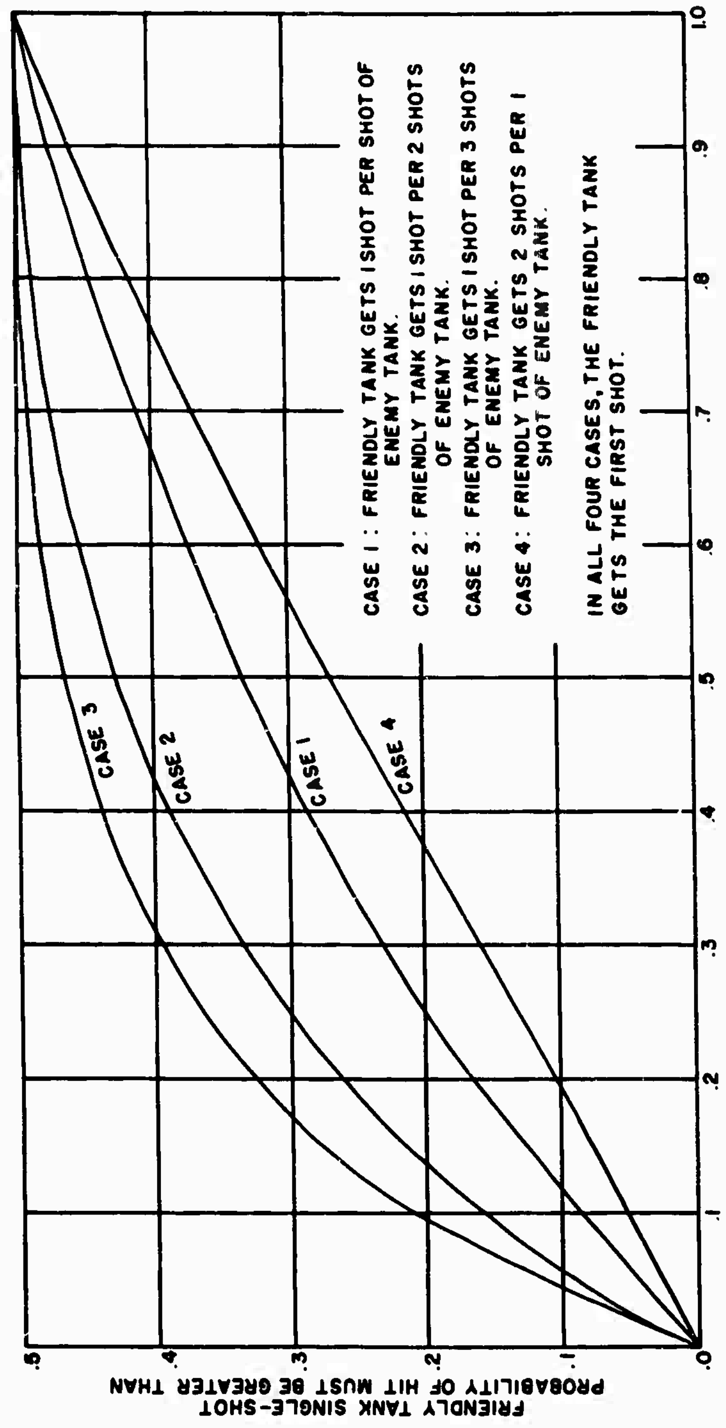

FIGURE 31. *Minimum Single-Shot Probability of Hit Which a Friendly Tank Must Have in Order to Have Greater Than .50 Survival Probability.*

(1) How much is required of any system of stabilization in terms of firing rate and accuracy for that system to give higher survival probabilities than halting to fire?

(2) Is the increased accuracy of using a range finder worth the shot it is necessary to give up to obtain one range reading?

(3) If a range finder is used, will the taking of more than one range reading result in increased survival probabilities?

(4) Are the survival probabilities of a stabilized tank greater if it halts after its first shot or if it continues to fire on the move?

(5) Is there some range at which a stabilized tank would do better firing as a non-stabilized tank?

While the answers to such questions are not conceptually difficult (for the case of a one-tank-vs-one-tank duel under the assumptions previously listed), they are rather laborious computationally.

Survival Probabilities: Derivations

The survival probabilities of a friendly tank in a duel with an enemy tank will be derived here.

Let F_i = the single-shot probability that the friendly tank will hit the enemy tank with the ith shot of the battle, provided that the duel continues to this shot.

E_i = an identical definition for the enemy tank.

C_i = the probability that the battle will continue to the $(i + 1)$st shot, which is equal to the probability that both tanks have survived all previous shots.

Let the conditions of the battle include the assumptions listed in the second paragraph of this section and consider the following:

Case 1

The friendly tank shoots first, and each tank then shoots in turn until one fails to survive.

This case describes the battle situation expected when the friendly tank is firing stabilized while the enemy tank halts to fire on the assumption that the stabilized tank has the same rate of fire (for example, a time advantage derived from the use of an automatic loader).

Table 15 shows, for each tank, the probability of that tank winning the battle after the ith shot and the probability that the battle will continue to the $(i + 1)$st shot.

Since the battle is to continue until one of the tanks wins, the probability that the friendly tank will survive (or kill the enemy tank) is the sum of the probabilities of this event for each shot. Thus, from Table 15 we have:

$$
\begin{aligned}
P_F &= F_1 + C_2F_3 + C_4F_5 + C_6F_7 + \dots \\
&= F_1 + [C_1(1 - E_2)]\,F_3 + [C_3(1 - E_4)]\,F_5 + [C_5(1 - E_6)]\,F_7 + \dots \\
&= F_1 + [(1 - F_1)(1 - E_2)]\,F_3 + [(1 - F_1)(1 - E_2)(1 - F_3)](1 - E_4)\,F_5 \\
&\quad + \{[(1 - F_1)(1 - E_2)(1 - F_3)(1 - E_4)](1 - F_5)\}(1 - E_6)\,F_7 \\
&\quad + \dots
\end{aligned}
$$

and since all shots of each tank are assumed to be fired with the same accuracy:

TABLE 15.

Tank Survival Probabilities

Case 1 of Section VIII

Shot No. $i =$	Probability that Friendly Tank Wins This Shot	Probability that Enemy Tank Wins This Shot	Probability that the Battle Continues $C_i =$
1	F_1	......	$(1 - F_1) = C_1$
2	......	C_1E_2	$C_1(1 - E_2) = C_2$
3	C_2F_3	......	$C_2(1 - F_3) = C_3$
4	......	C_3E_4	$C_3(1 - E_4) = C_4$
5	C_4F_5	......	$C_4(1 - F_5) = C_5$
6	......	C_5E_6	$C_5(1 - E_6) = C_6$
7	C_6F_7	......	$C_6(1 - F_7) = C_7$
etc.	etc.	etc.	etc.

$$F = F_1 = F_3 = F_5 = \text{etc.}$$
$$E = E_2 = E_4 = E_6 = \text{etc.}$$
$$P_F = F\left\{1 + (1 - F)(1 - E) + (1 - F)^2 (1 - E)^2 + (1 - F)^3 (1 - E)^3 + \ldots\right\}.$$

The term in the brackets is an infinite geometric series whose sum is $\dfrac{1}{1 - (1 - F)(1 - E)}$,

$$\text{so: } P_F = \frac{F}{1 - (1 - F)(1 - E)} \quad . \tag{12}$$

A plot of P_F as a function of E, for various values of F as parameter, is shown in Figure 32. The survival probability of the tank not getting the first shot (enemy tank) is

$$P_E = (1 - F)\, E\left\{1 + (1 - E)(1 - F) + (1 - E)^2 (1 - F)^2 + \ldots\right\}$$

$$P_E = \frac{(1 - F)\, E}{1 - (1 - F)(1 - E)} \tag{13}$$

Case 2

The friendly tank shoots first, the enemy tank shoots twice, the friendly tank shoots once again, the enemy tank shoots twice again, etc.

This case approximates the battle situation of a friendly nonmaneuvering stabilized tank versus an enemy tank halted to fire.

Following a development similar to that for Case 1, we obtain for this case

$$P_F = \frac{F}{1 - (1 - F)(1 - E)^2} \quad . \tag{14}$$

This expression is plotted in Figure 33.

Case 3

The friendly tank fires all shots with probability of hit F, and the enemy tank fires all shots with probability of hit E. The sequence of shots is as follows: the friendly tank

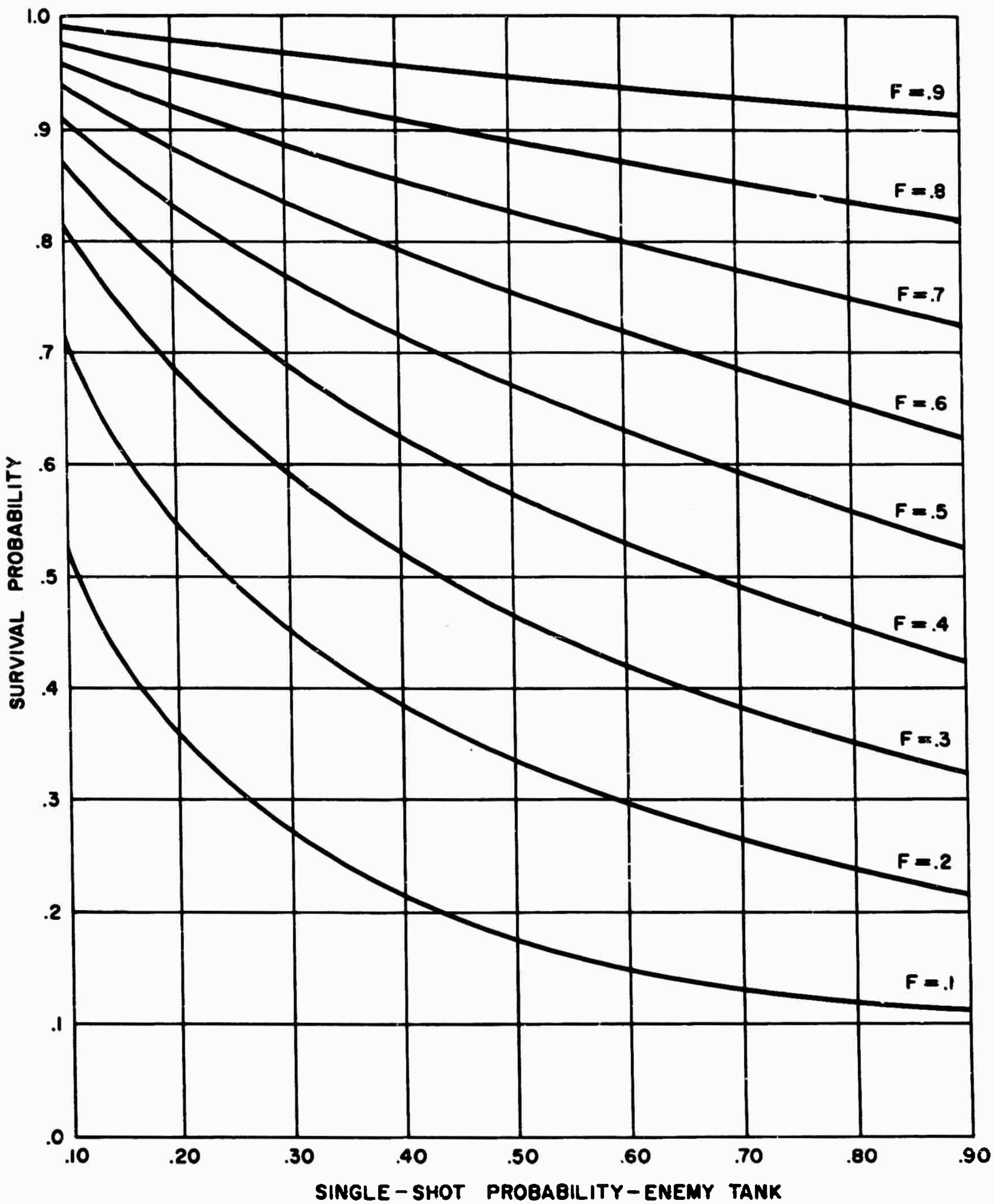

FIGURE 32. *Survival Probability in a Tank Battle of Friendly Tank Ratio of Firing Rates Equals 1 Shot for Friendly to 1 Shot for Enemy.*

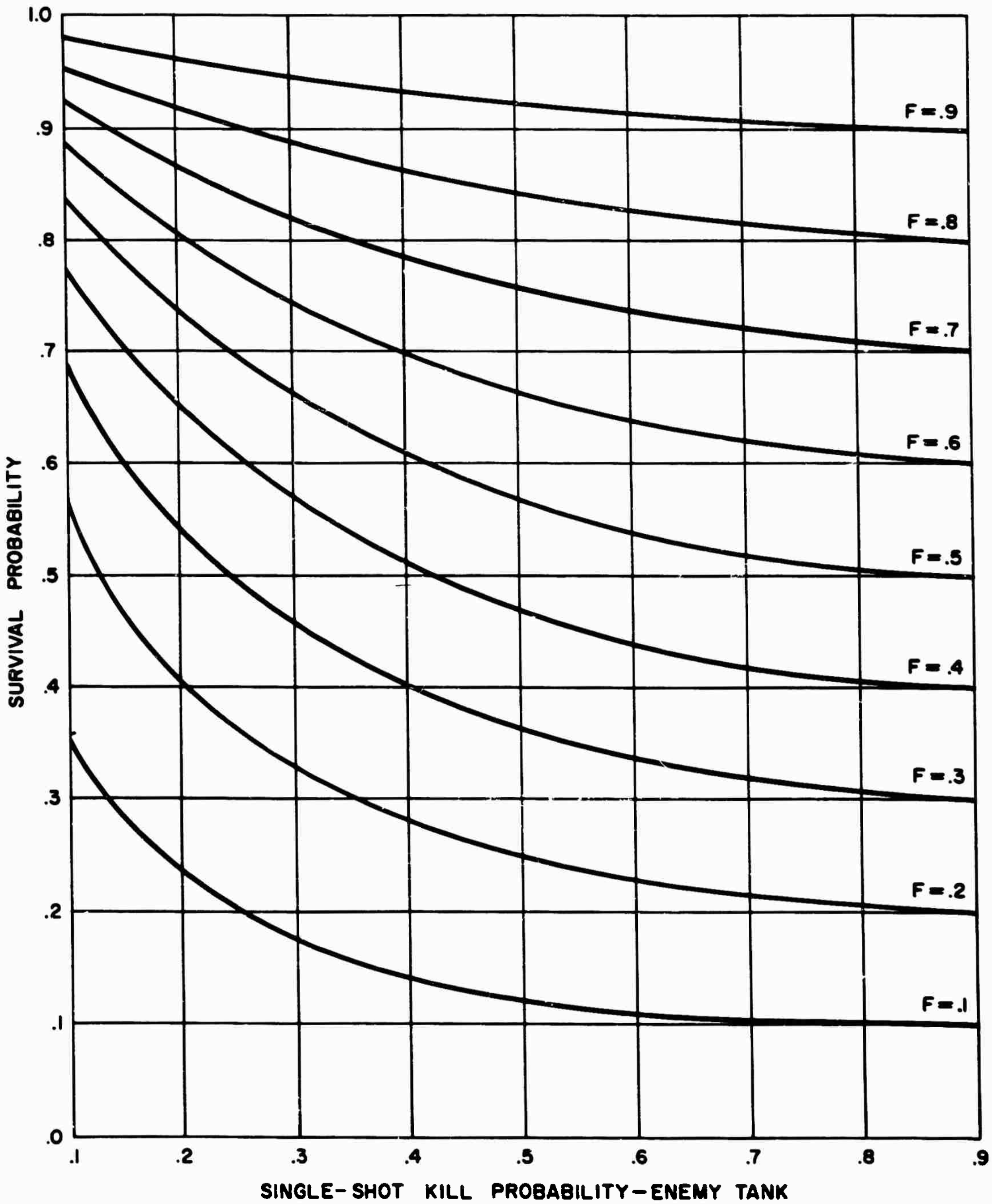

FIGURE 33. *Survival Probability in a Tank Battle of Friendly Tank Ratio of Firing Rates Equals 1 Shot for Friendly to 2 Shots for Enemy (Friendly Tank Fires First).*

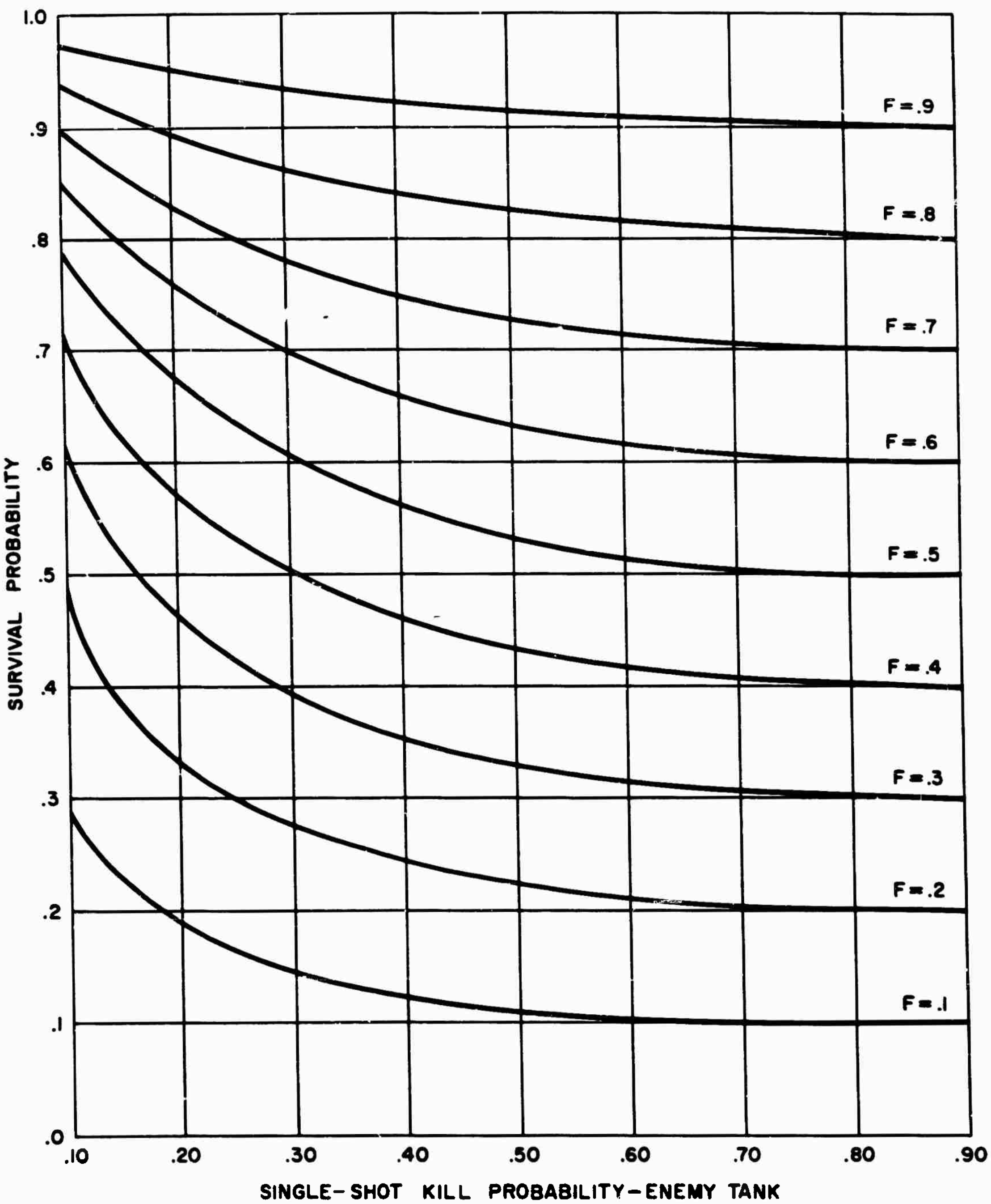

FIGURE 34. *Survival Probability in a Tank Battle of Friendly Tank Ratio of Firing Rates Equals 1 Shot for Friendly to 3 Shots for Enemy.*

gets the first shot, the enemy tank then shoots 3 shots; this is followed by one more shot by the friendly tank, then 3 more by the enemy tank, etc.

This case approximates the battle situation of a friendly stabilized maneuvering tank versus an enemy tank halted to fire.

Following a development similar to that for Case 1, we obtain for this case

$$P_F = \frac{F}{1 - (1-F)(1-E)^3} \quad . \tag{15}$$

This expression is plotted in Figure 34.

Case 4

The friendly tank shoots all shots with probability of hit F while the enemy tank shoots all shots with probability of hit E. The order of shooting is the first shot by the friendly tank, the next by the enemy tank, the next *two* by the friendly tank, the next one by the enemy, the next *two* by the friendly, etc.

This case is given for comparison purposes to show the advantage of getting both the first shot and a higher rate of fire.

Following a development similar to that for Case 1, we obtain for this case

$$P_F = \frac{F}{1 - (1-F)^2(1-E)} \left\{ 1 + (1-F)(1-E) \right\} \quad . \tag{16}$$

The restriction that all shots of the friendly tank (and/or the enemy tank) be fired with equal probability of hit is not necessary. The formula without such restriction merely becomes more cumbersome.

One may imagine a large variety of battles for other situations and, of course, derive similar formulas for each situation. For example, take that of Case 5:

Case 5

The friendly tank gets the first shot with probability of hit (F_1). The next two shots are by the enemy tank with probabilities of hit (E_2) and (E_3), where $E_3 > E_2$. The friendly tank shoots his next shot with probability of hit F_4 and each subsequent shot with probability F_6. The enemy tank shoots alternate shots (after the friendly tank's second), all with probability E_5. This case is given as an approximation of the battle situation wherein the friendly tank fires stabilized for the first shot only and then halts to fire. Each tank uses burst-on-target for shots subsequent to the first after halting. Thence:

$$\begin{aligned} P_F = {} & F_1 + (1-F_1)(1-E_2)(1-E_3)F_4 + (1-F_1)(1-E_2)(1-E_3)(1-F_4)(1-E_5)F_6 \\ & + (1-F_1)(1-E_2)(1-E_3)(1-F_4)(1-E_5)^2(1-F_6)F_6 \\ & + (1-F_1)(1-E_2)(1-E_3)(1-E_4)(1-E_5)^3(1-F_6)^2F_6 \\ & + \ldots . \\ = {} & F_1 + (1-F_1)(1-E_2)(1-E_3) \\ & \left\{F_4 + (1-F_4)(1-E_5)[1 + (1-E_5)(1-F_6) + (1-E_5)^2(1-F_6)^2 + \ldots .]F_6\right\} \\ = {} & F_1 + (1-F_1)(1-E_2)(1-E_3)\left\{F_4 + \frac{(1-F_4)(1-E_5)F_6}{1-(1-E_5)(1-F_6)}\right\} \end{aligned} \tag{17}$$

Case 6

It is first desired to compare each situation as given in Table 13 with the one in which each tank has an equal chance of getting the first shot but otherwise fires as in Case 1. This is approximately the situation when both tanks halt to fire. The survival probability of the friendly tank is:

$$P_F = (.5)\left\{\frac{F}{1-(1-F)\,(1-E)}\right\} + (.5)\left\{\frac{(1-E)\,F}{1-(1-F)\,(1-E)}\right\} ,$$

$$P_F = \frac{\tfrac{1}{2}F\,(2-E)}{1-(1-F)\,(1-E)} . \qquad (18)$$

For purposes of exposition, this situation will be referred to as Case 6.

It will be assumed that the single-shot probability of hit remains constant throughout the battle. This assumption is not warranted in several instances: for example, whenever it is possible to use the burst-on-target technique for reducing the range error. However, while the assumption is introduced here for simplicity it may easily be removed, as indicated in Case 5.

It is now desired to determine how much reduction in single-shot probability of hit it is possible to sacrifice in order to obtain the advantage of getting the first shot while still retaining the same survival probability as in Case 6. Case 6 will be compared with each of those listed in Table 13. In each of the following comparisons, lower-case letters will be used for Case 6 probabilities:

Case 6 vs. Case 1

Since survival probabilities are to be at least equal,

$$\frac{\tfrac{1}{2}f\,(2-e)}{1-(1-f)\,(1-e)} < \frac{F}{1-(1-F)\,(1-E)} .$$

Solving for F and simplifying:

$$F > \frac{f\,(2-e)\,E}{e\,(2-f)+f\,(2-e)\,E}$$

and if $e = f$, then

$$F > \frac{E}{1+E} . \qquad (19)$$

Thus, if the enemy tank has a probability of hit $E = 1$, then the survival chances of the friendly tank are increased if his probability of hit is greater than ½, provided he is certain of getting the first shot. Similarly, if the enemy's probability of hit is ½, then the survival probability of the friendly tank is increased if his probability of hit is greater than ⅓, provided that he is certain to get the first shot.

It is interesting to note that, if both tanks have equal single-shot probability of hit under Case 6, the accuracy which is required by the friendly tank in order to increase survival chances over that which existed for Case 6 is a function only of the accuracy of the enemy tank under Case 1.

Case 6 vs. Case 2

Again, since survival probabilities are to be at least equal in the two cases,

$$\frac{\tfrac{1}{2} f (2 - e)}{1 - (1 - f)(1 - e)} < \frac{F}{1 - (1 - F)(1 - E)^2} \ .$$

Solving for F and simplifying:

$$F > \frac{f (2 - e) - f (2 - e)(1 - E)^2}{2\left[1 - (1 - f)(1 - e)\right] - f (2 - e)(1 - E)^2}$$

and if $e = f$, then

$$F > \frac{1 - (1 - E)^2}{2 - (1 - E)^2} \ . \qquad (20)$$

While the required F ($> \tfrac{1}{2}$) is the same for Case 2 as for Case 1 when $E = 1$, it is larger for any other accuracy of the enemy. Thus for $E = \tfrac{1}{2}$, F must be equal to, or greater than, .428 rather than .333 in order to retain at least the same survival chance as in Case 6. This is to be expected because the lower rate of fire of the friendly tank must be made up with more accuracy.

Case 6 vs. Case 3

As before, the requirement that survival probabilities in Case 3 be at least equal to those in Case 6 means that:

$$\frac{\tfrac{1}{2} f (2 - e)}{1 - (1 - f)(1 - e)} < \frac{F}{1 - (1 - F)(1 - E)^3} \ .$$

After solving for F and simplifying:

$$F > \frac{f (2 - e)\left[1 - (1 - E)^3\right]}{2 - 2(1 - f)(1 - e) - f (2 - e)(1 - E)^3}$$

and if $e = f$

$$F > \frac{1 - (1 - E)^3}{2 - (1 - E)^3} \ . \qquad (21)$$

As in the situation of Case 2 vs. Case 6, the required value of F for Case 3, which is necessary in order to maintain at least equal survival probability to Case 6, is increased over that of Case 1. If (under Case 3) $E = \tfrac{1}{2}$, then F must be equal to, or greater than, .467 in order to retain at least the same survival chance as in Case 6.

IX. CONCLUSIONS AND RECOMMENDATIONS

The results of this study indicate that future tank development should consider the following approach:

(1) Stabilization of the sight as tightly as possible in azimuth and elevation.

(2) Stabilization of the gun in azimuth and elevation only to the extent required for following the sight gyro.

(3) Use of the three-switch firing arrangement in conjunction with limited gun stabilization.

It is recommended that this arsenal conduct further studies along the following lines:

(1) Determination of the angular velocity of an unstabilized gun in azimuth and elevation to ascertain whether there is necessity for limited gun stabilization.

(2) Determination of the optimum dimensions of the firing region (under the three-switch firing proposal).

(3) Carry on investigations to measure the magnitude of the sight error.

(4) Carry on additional investigations relative to the magnitude of the gun error.

It is further recommended that operations research and analysis functions of the Department of the Army, including those in the Ordnance Corps, undertake the following additional programs:

(1) Detailed study of the tactical uses of tanks and the battle situations in which tanks become involved.

(2) Investigation of more detailed and elaborate tank duels.

X. REFERENCE LIST

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